

On the hierarchy problem of particle physics

Deep Bhattacharjee ¹

¹Electro Gravitational Space Propulsion Laboratory

October 31, 2023

Abstract

This paper attempts to delve into the hierarchy problem of particle physics taking into consideration the Calabi-Yau manifolds and D(p)-Branes.

On the hierarchy problem of particle physics

Deep Bhattacharjee

Abstract:

This paper attempts to delve into the hierarchy problem of particle physics taking into consideration the Calabi-Yau manifolds and $D(p)$ – Branes.

Introduction:

Gravity is pervasive. Gravity is everywhere and it spans over a vast distance in cosmic scales but still it is one of the weakest of the 4-fundamental forces of nature. Recent discoveries in supersymmetry and superstrings have unfolded the surprising nature of the gravity in the compactified extra-dimensions. However, there are still important effects that gravity can induce if the dimensions (or extra-dimensions) are larger in scale. M-theory and F-theory are successful in the scheme of unification where especially in F-theory, the unification of gauge bosons have been considered beautifully from the respective cycles of elliptic fibrations. TypeIIB string theory which is both S and T-dual to itself contains closed strings and these closed strings of mass spectrum 0 and spin 2 gives birth to gravity. Gravity always generates from the first state of the harmonic oscillation along with a dilaton. Although current technologies lags to unify gravity with electromagnetism and strong-weak nuclear forces, different models have evolved like the ADD-model, Randall-Sundrum model thereby paving the way for the solving of the 'Hierarchy problem' in particle physics. Gravity, however, is very difficult to detect in Atom-Colliders as they can easily escape to other dimensions thereby proving that Graviton (particles of gravity) are capable of inter-dimensional travel. To the further extent, gravity being relativistically a geometry of space-time has found its place in the complex manifolds of hidden extra-dimensions. The monopole nature of gravity which originates in higher dimensions due to string anomalies is fascinating to compute. This paper focuses mostly on the nature of gravity and how it behaves in higher dimensions with a probable attempt to explain the hierarchy problem.

Keywords:

Monopole, Electromagnetism, Superstrings, $D(p)$ -Brane.

Interpretation:

1. Construction:

$D(p)$ – Branes cannot confine gravity as because they are not open strings rather closed strings and is spread through 10-spatial dimensions. $D(4)$ one being our known 4-dimensional universe. In case of $D(5)$ – Brane, gravity gets coupled with electromagnetism as per Kaluza Klein (KK) – theory and unification of the two forces occurs. If a torus T^2 is considered having its surface contour closed, then the density of the flux lines becomes zero. This can be stated as [1, 2, 3],

$$\oint_{\text{flux}} T^2 = 0$$

According to Gauss law, if the flux which spreads over the surface or passing through the surface is not equal to zero, that is,

$$\oint_{\text{flux}} T^2 \neq 0$$

Whereas magnetic monopoles exist as,

$$\oint_{\text{flux}} \mathbf{B} ds = 4\pi\rho_m$$

$\exists\rho_m$ being the field-flux density

Gravity is always attractive, or it can be said that gravity is always unipolar as because there are no such poles ($N - S$). Now, either $T^n = (S^1)^2$ where $n = 2$ in case of lower dimensions or $T^n = (S^1)^n$ in case of n – higher dimensions. If monopole exists, the equation,

$$\oint_{\text{flux}} \mathbf{B} ds = 4\pi\rho_m \equiv \nabla \cdot \mathbf{B} = 4\pi\rho_m$$

As T^2 or T^n consists of major and minor radius, therefore, an operator $\mathcal{U}$ acts on the minor radius to slice it vertically thereby forming an open cylinder $\mathcal{C}$. The cylinder can be made infinite by dropping one of its origins as $(\mathbb{C}^\times = \mathbb{C} \setminus \{0\})$, $\mathbb{C}$ being complex manifold. This will provide useful in later sections where Calabi-Yau manifolds of vanishing Ricci curvature is provided by extending the complex plane of the cylinder. The KK-theory would hold true in $\tilde{g}_{55}$ metric with the normal vector N of T^2 or T^n relies with B as $(N * B \cos \theta)$.

2. Dimensional anomaly and $D(p)$ -Branes:

Two different dimensions are taken as $D(a)$ and $D(b)$. If the cylinder C spans over these two dimensions with one end in $D(a)$ and other end in $D(b)$ and if we assume C as a flux-tube with flux lines entering from one end and exiting from other end without any loss of dissipation, then North Pole lies in $D(a)$ while South Pole lies in $D(b)$ (or vice versa). Then, there originates a Unipolar gravity such that the magnitude $\mathbf{M}$ of the gravity is,

$${}^n_{D(a)}\mathbf{M} \approx {}^n_{D(b)}\mathbf{M}$$

$\exists n$ being the generalized dimensions

Now, it is stated before that $\nabla \cdot \mathbf{B} = 4\pi\rho_m$ and if charge density parameter ρ_m = mass/unit volume and when the volume is 0, then $\nabla \cdot \mathbf{B} = \infty$. This establishes the relation clearly that dimension is inversely proportional to the volume or,

$$D \propto 1/V$$

Considering 2 magnitudes of flux, γ^1 and γ^2 such that $\gamma^2 \gg \gamma^1$, then if the dimensions $\gamma^1_D \gg \gamma^2_D$, then the dimensionality relation established as,

$$\gamma^2/\gamma^1 \rightarrow \gamma^2_D/\gamma^1_D$$

So, $\mathbf{B}$ tends to increase if the circumference of the cylinder C , i.e.,

$$2\pi R \xrightarrow{\text{approaches}} 0 \text{ as } D \propto 1/V$$

Lorentz Generators are essential for the computations of dimensions (or critical dimensions). The generator satisfies an equation as such [4, 5],

$$[L^{-i}, L^{-j}] = \{\emptyset * (n(P-2)/8) + 1/n(2\alpha_{NS} - (P-2)/8) - n\}$$

Here, ' α_{NS} ' is the excitation mode, P is the dimensions, $\emptyset$ is the composite affine parameter related to mode ' α_{NS} '. Here $[L^{-i}, L^{-j}] = 0$ and to prove $\{\emptyset * (n(P-2)/8) + 1/n(2\alpha_{NS} - (P-2)/8) - n\} = 0$, we set $(P-2)/8 = 1$, then $P = 10$ just as $D(P) = D(10)$ as we see at the beginning of the paper. Then, for the RHS to be 0, let $2\alpha_{NS} - (P-2)/8 = 0$, then $\alpha_{NS} = 0.5$.

2.1 Extra dimensions and Calabi-Yau manifolds

If the extra dimensions are compactified, then they are curled up in a circle and they take the value $2\pi R + \ell \approx \delta^{-1}$. R being the radius of the curled-up dimensions (small extra dimensions) and δ^{-1} being the affine parameter representing $D(p)$ Branes. The Lagrangian of the lowest dimensional ($D(1)$ -Brane) can be given as [3, 4, 5, 6, 7],

$$\mathcal{L} = m \left(\frac{1}{2} \dot{X}^2 - Kgh \right)$$

This has a factor called K which is the spring constant and therefore $K = \sqrt[2]{\omega}$. This factor of $\mathcal{L}$ when recoils then exert an opposite force $-F$ as per the Newton's 3rd law which we will come in discussion very soon.

Let us parametrize the 6-dimensional Calabi-Yau manifolds (fourfolds) as ${}^{\sigma}_6\beta$ and ${}^{\tau}_6\beta$. Now, there are 2 ends of this CY manifolds, one end is finite containing the curled-up dimensions whereas, the origin being dropped as per the equations ($\mathbb{C}^\times = \mathbb{C} \setminus \{0\}$), this infinite end contains the 4 large space-time dimensions of $D(4)$ – Brane.

If the T^n torus at the finite end of the CY manifold contains a Δ factor as ΔT^n and if the limit taken as,

$$\lim_{\Delta \rightarrow 0} \Delta T^n$$

Then a singularity is achieved at the ends of ${}^{\sigma}_6\beta$ and ${}^{\tau}_6\beta$. If the ends of ${}^{\sigma}_6\beta$ and ${}^{\tau}_6\beta$ are facing each other and the $D - 1$ Strings which originates from the singularities of the collapsing ends of ${}^{\sigma}_6\beta$ and ${}^{\tau}_6\beta$, and as $D - 1$ Brane is a kind of monopole, then when they comes in contact with each other the opposite force $-F$ of $\mathcal{L}$ comes in contact, they recoils and curled up to form a closed string graviton. This recoil force is enough to make the strings detached from the curled up finite ends of CY fourfold and travels towards the infinite ends representing large 4 space-time dimensions. This clearly explains why gravity is 10^{24} times weaker compared to the four fundamental forces and delves into the hierarchy problem smoothly.

Now the T^2 or T^n torus which degenerates satisfies a cycle as,

$$\tau_{\epsilon^{11}} = a\tau + b/c\tau + d$$

Where ϵ^{11} when the cycle of the minor radius ' r ' satisfies $\oint \epsilon_r^{11} = 1$ and ϵ^{22} when the cycle of the major radius R satisfies $\oint \epsilon_R^{22} = \tau$.

3. Concluding remarks:

Every possible aspects of the canonical formation of gravity has been thoroughly discussed and presented with the help of theories and mathematics leading to the calculation of the compactified dimensions, to magnetic monopoles analogues to unipolar gravity from KK-excitations and the topological complex manifold of a Kähler type (or calabi-Yau) has been discussed with torus degeneration leading to the movement of gravity from the finite to infinite ends thereby solving the 'Hierarchy Problem' of particle physics.

4. References:

- 1] Irodov, I. E. (2020, September 1). *Basic Laws of Electromagnetism*.
- 2] Kamberaj, H. (2022, May 19). *Electromagnetism*. Springer.
- 3] Borisenko, A. I., & Tarapov, I. E. (2012, August 28). *Vector and Tensor Analysis with Applications*. Courier Corporation.
- 4] McMahon, D. (2009). *String theory demystified*. New York: McGraw-Hill.
- 5] Bhattacharjee, D. (2022, August 16). *Generalization of Quartic and Quintic Calabi – Yau Manifolds Fibered by Polarized $K3$ Surfaces*.
<https://doi.org/10.21203/rs.3.rs-1965255/v1>
- 6] Bhattacharjee, D. (2022, July 15). M-Theory and F-Theory over Theoretical Analysis on Cosmic Strings and Calabi-Yau Manifolds Subject to Conifold Singularity with Randall-Sundrum Model. *Asian Journal of Research and Reviews in Physics*, 25–40. <https://doi.org/10.9734/ajr2p/2022/v6i230181>
- 7] Arkani-Hamed, N., Dimopoulos, S., & Dvali, G. (1999). Phenomenology, astrophysics, and cosmology of theories with submillimeter dimensions and TeV scale quantum gravity. *Physical Review D*, 59(8). doi: 10.1103/physrevd.59.086004