Asymmetric Bidirectional Quantum Teleportation employing hexa-modal Entangled Coherent States as the Quantum Channel

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Abstract

A Bidirectional Quantum Teleportation protocol between two parties (Alice and Bob) is investigated in which Alice possesses arbitrary entangled coherent state and Bob preserve unknown superposed coherent state via a quantum channel (hexa- modal entangled coherent states). This assymeteric bidirectional protocol is worked out in continuous variable (coherent states) regime.

TABLE I

HERALDED STATE VECTORS $|\emptyset\rangle_{3,4,,6}$ FOR PHOTON-COUNTS (p, p', p'') AND (q, q, q'').HERE, U_i^R IS A (ROTATION) UNITARY OPERATOR WHICH ROTATES ABOUT Z AXIS. IN Z-ROTATION, $U^R\left(\frac{\pi}{2}\right) = \begin{pmatrix} e^{i\pi/2} & 0 \\ 0 & e^{-i\ \ /2} \end{pmatrix}$ ANDP IS PHASE SHIFTER $P(\psi)$= EXP(-IA†AΨ) SUCH THAT $|\alpha\rangle \rightarrow P(\psi)\mathrm{D}(\alpha)|0\rangle \rightarrow |e^{i\psi}\alpha\rangle,\ take\ \psi = \pi,\ |\alpha\rangle = |-\alpha\rangle$, AND I IS AN IDENTITY OPERATOR. IF PHOTON COUNTSp, p', p''AND q, q', q''ARE ZERO, ABQT FAILS.

$p/\ q'$	$p'/\ q'$	$p''/\ q''$	**Heralded State-vectors corresponding to $p, p', p'' \neq 0$ $q, q', q'' = 0$**	**Unitary (gate) operation**		**Heralded State-vectors corresponding to $q, q', q'' \neq 0$ $p, p', p'' = 0$**	**Unitary (gate) operation**	
				Alice Lab	**Bob Lab**		**Alice Lab**	**Bob Lab**
ODD	ODD	ODD	$(a_0 \lvert\alpha,\alpha\rangle + a_1\lvert-\alpha,-\alpha\rangle)_{3,4} \otimes (b_0\lvert\alpha\rangle - b_1\lvert-\alpha\rangle)_6$	U_6^R	$I_3 \otimes I_4$	$(a_0 \lvert-\alpha,-\alpha\rangle + a_1\lvert\alpha,\alpha\rangle)_{3,4} \otimes (b_0\lvert-\alpha\rangle - b_1\lvert\alpha\rangle)_6$	$U_6^R \otimes P_6$	$P_3 \otimes P_4$
ODD	ODD	EVEN	$(a_0 \lvert\alpha,\alpha\rangle + a_1\lvert-\alpha,-\alpha\rangle)_{3,4} \otimes (b_0\lvert\alpha\rangle + b_1\lvert-\alpha\rangle)_6$	I_6	$I_3 \otimes I_4$	$(a_0 \lvert-\alpha,-\alpha\rangle + a_1\lvert\alpha,\alpha\rangle)_{3,4} \otimes (b_0\lvert-\alpha\rangle + b_1\lvert\alpha\rangle)_6$	U'_6	$P_3 \otimes P_4$
ODD	EVEN	ODD	$(a_0 \lvert\alpha,\alpha\rangle - a_1\lvert-\alpha,-\alpha\rangle)_{3,4} \otimes (b_0\lvert\alpha\rangle - b_1\lvert-\alpha\rangle)_6$	U_6^R	$U_3^R \otimes U_4^R$	$(a_0 \lvert-\alpha,-\alpha\rangle - a_1\lvert\alpha,\alpha\rangle)_{3,4} \otimes (b_0\lvert-\alpha\rangle - b_1\lvert\alpha\rangle)_6$	$U_6^R \otimes P_6$	$(U_3^R \otimes U_4^R)\ (P_3 \otimes P_4)$
EVEN	ODD	ODD	$(a_0 \lvert\alpha,\alpha\rangle - a_1\lvert-\alpha,-\alpha\rangle)_{3,4} \otimes (b_0\lvert\alpha\rangle - b_1\lvert-\alpha\rangle)_6$	U_6^R	$U_3^R \otimes U_4^R$	$(a_0 \lvert-\alpha,-\alpha\rangle - a_1\lvert\alpha,\alpha\rangle)_{3,4} \otimes (b_0\lvert-\alpha\rangle - b_1\lvert\alpha\rangle)_6$	$U_6^R \otimes P_6$	$(U_3^R \otimes U_4^R)(P_3 \otimes P_4)$
EVEN	EVEN	EVEN	$(a_0 \lvert\alpha,\alpha\rangle + a_1\lvert-\alpha,-\alpha\rangle)_{3,4} \otimes (b_0\lvert\alpha\rangle + b_1\lvert-\alpha\rangle)_6$	I_6	$I_3 \otimes I_4$	$(a_0\lvert-\alpha,-\alpha\rangle + a_1\lvert\alpha,\alpha\rangle)_{3,4} \otimes (b_0\lvert-\alpha\rangle + b_1\lvert\alpha\rangle)_6$	P_6	$P_3 \otimes P_4$
EVEN	EVEN	ODD	$(a_0 \lvert\alpha,\alpha\rangle + a_1\lvert-\alpha,-\alpha\rangle)_{3,4} \otimes (b_0\lvert\alpha\rangle - b_1\lvert-\alpha\rangle)_6$	U_6^R	$I_3 \otimes I_4$	$(a_0\lvert-\alpha,-\alpha\rangle + a_1\lvert\alpha,\alpha\rangle)_{3,4} \otimes (b_0\lvert-\alpha\rangle - b_1\lvert\alpha\rangle)_6$	$U_6^R \otimes P_6$	$P_3 \otimes P_4$
EVEN	ODD	EVEN	$(a_0 \lvert\alpha,\alpha\rangle - a_1\lvert-\alpha,-\alpha\rangle)_{3,4} \otimes (b_0\lvert + b_1\lvert-\alpha\rangle)_6$	I_6	$U_3^R \otimes U_4^R$	$(a_0\lvert-\alpha,-\alpha\rangle - a_1\lvert\alpha,\alpha\rangle)_{3,4} \otimes (b_0\lvert-\alpha\rangle + b_1\lvert\alpha\rangle)_6$	P_6	$(U_3^R \otimes U_4^R)\ (P_3 \otimes P_4)$
ODD	EVEN	EVEN	$(a_0 \lvert\alpha,\alpha\rangle - a_1\lvert-\alpha,-\alpha\rangle)_{3,4} \otimes (b_0\lvert\alpha\rangle + b_1\lvert-\alpha\rangle)_6$	I_6	$U_3^R \otimes U_4^R$	$(a_0\lvert-\alpha,-\alpha\rangle - a_1\lvert\alpha,\alpha\rangle)_{3,4} \otimes (b_0\lvert-\alpha\rangle + b_1\lvert\alpha\rangle)_6$	P_6	$(U_3^R \otimes U_4^R)\ (P_3 \otimes P_4)$

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***Abstract*—We report a protocol in which Asymmetric Bidirectional Quantum Teleportation between two parties (Alice and Bob) is witnessed by employing a pair of one four coherent-modes GHZ-like state and one coherent-modes Bell-pair state, i.e, hexa-modal cluster-type entangled coherent state as the quantum channel. A heralded detection of photons in laboratories of Alice and Bob followed by classical communications of photon-numbers and local unitary (gate) operations therein, perfectly, demonstrates the protocol with 50% probability.**

***Index Terms*—Bidirectional Quantum Teleportation, Beam splitter, Coherent state, Entangled coherent states, Photon number resolving detector.**

I. INTRODUCTION

QUANTUM Teleportation is one of the foundational and fundamental earliest works in Quantum Information Processing discovered by Bennet et al. in 1992 [1], termed as standard quantum teleportation (SQT) in discrete-variable (DV) regime. SQT is experimentally realized first in 1997 [2], and now, is progressed successfully to be a mature quantum technology [3]. Braunstein and Kimble [4] have developed a scheme to teleport the wave function of single mode electromagnetic field using bi-modal vacuum squeezed state, which is verified in seminal experiment due to Furusawa et al[5].

Optical Coherent states are one of the promising candidates for continuous variable (CV) quantum information processing because of its feasible experimental preparation by LASERs and an easy manipulation by linear optical devices such as beam splitter, phase shifter and photon detector. Cochrane et al. [6] in 1999 has introduced a logical-mode encodings based on superposed coherent states namely, even (symmetric) coherent states [7]. Relaxing the orthogonality condition of multi-photon even and odd coherent states Ralph et al. in 2003 [8] have shown that coherent states, itself, may be employed for encoding logical-qubits via $|0\rangle_L \rightarrow |\alpha\rangle$ and $|1\rangle_L \rightarrow |-\alpha\rangle$, α is here, taken real, where $|\alpha\rangle$ and $|-\alpha\rangle$ are π-phase apart optical coherent states.

van Enk and Hirota [9], have developed the first fundamental protocol in which a quantum-information encoded in superposed coherent states is teleported utilizing only linear optical devices such as beam splitters, phase shifters and photon number resolving detectors to non-locally stationed Bob via bi-modal maximally entangled coherent states as quantum channel. Wang [10] have generalized the van Enk and Hirota's protocol in which a quantum-information encoded in multi-modal entangled coherent states is transmitted by multi-modal entangled coherent states [11]. In 2007, Prakash et al. [12] brought out an improvement on Wang's protocol by introducing a finer consideration in the classification of the results of photon-counting which may only be used to infer local unitary operations. Historically, SQT scheme [1], finds 'first' generalization by inserting third party Charlie in the protocol. Charlie is intertwined with parties Alice and Bob by sharing three particle (qubits) entanglement prepared in maximally entangled GHZ state. Since the third-party Charlie, here controls the perfect receiving of quantum state at Bob's lab, this generalization of SQT may be named as Controlled Quantum teleportation (CQT) [13]. Unidirectional CQT may finds myriad variants by involving 'many parties', multi qubit information-states and invoking security-issues by developing novel communication protocols in DV regime [14]–[17].

Notably, CQT in CV-regime is worked out by Pandey et al. [18] using optical coherent states for quantum information-state and GHZ-entangled coherent states as quantum channel. The 'Bi-directional Quantum Teleportation (BQT) protocol has appeared in Huelga et al. [19], who have, while investigating implementation of arbitrary unitary operation upon distantly located quantum system, proven that the required resources for the same are those of implementing BQT. In 2013 a variation of BQT, namely, Bidirectional 'controlled' quantum teleportation (BCQT) is proposed in DV-regime by Zha et al.[20], of which generalization is proposed by Shukla et al. [21]. BCQT is a three-party scheme wherein parties are intertwined by correlated multi-particle maximally or mixed entangled states and communication of quantum-information encoded either in single qubits or in entangled qubits (symmetric or antisymmetric) possessed by sender (receiver) are exchanged with the assistance of third party acting as 'controller'. A spurt of interests has been witnessed by many research works on BQT [22]–[24] and its ramification in DV–regime on the basis of 'to be teleported single (multi) qubit information-state' at sender (receiver) end and, also various kinds of multi-qubit entangled state utilized as quantum channel [25], [26]. BCQT may be employed to

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transport quantum states in a quantum network. Communication in quantum network, indispensably, necessitates the diverse kind of exchange of quantum states possessed by parties such as asymmetric bidirectional quantum teleportation [27]–[29], cyclic quantum teleportation [30], [31], cyclic 'controlled' quantum teleportation [32]–[34], multidirectional quantum teleportation in DV-regime [35], [36].

In 2021 Pandey et al. [37] proposed a symmetric BCQT in CV-regime utilizing linear optical devices in which information-states at sender(receiver) station are encoded in superposed optical coherent states and penta-modal cluster-type entangled coherent states is employed as quantum channel. Pertinently, it may be noted that a scheme for symmetric BQT by using optical coherent state is put forth by Aliloute et al. [38] in 2021 wherein tri-modal entangled coherent states is utilized as quantum channel. Furthermore, a work on BQT of even and odd coherent states [30] with multimodal entangled coherent–states as a quantum channel is worked out but by using overhead demand of trigger modes[39]a coherent-states version of a scheme due to Kiktenko [40]. Aliloute et al. [38] committed a serious blunder by envisaging an improbable 'action' where the quantum-information possessed by nonlocally stationed Alice and Bob are mixed by allowing them to incident on beam splitter (see Fig. 1 in [38]). That is to say, no genuine Asymmetric Bi-directional Quantum Teleportation (ABQT) scheme in CV-regime exist in literature, which necessitates the present work.

In the following investigation an ABQT protocol is proposed by employing hexa-modal cluster-type entangled coherent states as quantum channel, which may be seen as the tensor-product of a pair of one four coherent-mode GHZ-like state and one coherent- mode Bell-pair states. Here, the information-quantum state possessed by Alice is entangled coherent states and Bob keeps superposed even coherent states and the two parties Alice and Bob are nonlocally stationed and intertwined by distributing modes from quantum channel. The letter is structured in the following Sections. Section (II) describes ABQT scheme in detail. Section (III) provides the probability and fidelity consideration.

II. ASYMMETRIC BIDIRECTIONAL QUANTUM TELEPORTATION

In the teleportation scheme worked out, here, Alice wish to teleport the entangled coherent state,

$$|\psi\rangle_{AA'} = N_{AA'}(a_0|\alpha,\alpha\rangle + a_1|-\alpha,-\alpha\rangle)_{AA'}, \qquad (1)$$

where $N_{AA'} = [|a_0|^2 + |a_1|^2 + 2e^{-4|\alpha|^2} Re\,(a_0 a_1^*)]^{-1/2}$, is a normalization constant and α is taken to be real for simplicity. Simultaneously, Bob has to transmit a single mode encoded in even coherent state [9]

$$|\psi\rangle_B = N_B(b_0|\alpha\rangle + b_1|-\alpha\rangle)_{B,}, \qquad (2)$$

where, $N_B = [|b_0|^2 + |b_1|^2 + 2e^{-2|\alpha|^2} Re(b_0 b_1^*)]^{-1/2}$, is a normalization constant.

Since Alice's 'to be transmitted' quantum information state is bi-modal entangled coherent state and that from Bob to Alice is mono-modal even coherent states, this teleportation protocol may, tacitly, be termed as Asymmetric Bidirectional Quantum Teleportation (ABQT). In order to accomplish ABQT, we employed hexa-modal entangled coherent states,

$$\begin{aligned} |\psi\rangle_{1,2,3,4,5,6} = &\left[2\sqrt{1+x^4+x^8+x^{12}}\right]^{-1} \\ &(|\alpha,\alpha,\alpha,\alpha\rangle + |-\alpha,-\alpha,-\alpha,-\alpha\rangle)_{1,2,3,4} \\ &\otimes(|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle)_{5,6} \end{aligned} \qquad (3)$$

where $x \equiv e^{-|\alpha|^2}$, as a quantum channel in which the modes 1,2,6 are taken in possession of Alice's lab and modes 3, 4 and 5 belong to Bob 's lab, respectively. Obviously (3) is tensor-product of a pair of four coherent-modes GHZ-like state and one coherent-mode Bell-pair state demonstrating that all modes are not entangled.

The composite state of total system can be written as,

$$|\phi\rangle_{A,A',B,1,2,3,4,5,6} \propto |\psi\rangle_{AA'} \otimes |\psi\rangle_B \otimes |\psi\rangle_{1,2,3,4,5,6} \qquad (4)$$

ABQT protocol may be outlined in following steps:

Step1- Alice mixes modes A *and* A' with quantum channel's modes 1and 2 respectively, using a 'symmetric beam splitter with phase shifter' (BPS-1) and BPS-2 and perform photon-counting measurements at output modes 7,8,9,10 using detectors $D_{7,8,9,10}$, respectively. Similarly, Bob mixes mode B with 5 using BPS-3 and detects photons at output modes (11,12) using detectors $D_{11,12}$. Photon-counts are communicated by Alice to Bob and vice versa.The BPS is characterized by the transformation Ɓ defined as,

$$Ɓ|\alpha,\beta\rangle_{xy} = (|(\alpha+\beta)/\sqrt{2}\rangle)_u, (|(\alpha-\beta)/\sqrt{2}\rangle)_v \qquad (5)$$

where $|\alpha,\beta\rangle_{xy}$ is bi-modal coherent states. Applying (5) with Step-1,(4) yields,

$$\begin{aligned} &|\phi\rangle_{7,8,9,10,11,12,2,3,6} \propto \\ &a_0b_0\big(|\sqrt{2}\alpha,0\rangle_{7,8}|\sqrt{2}\alpha,0\rangle_{9,10}|\sqrt{2}\alpha,0\rangle_{11,12}|\alpha,\alpha,\alpha\rangle_{346} + \\ &|\sqrt{2}\alpha,0\rangle_{7,8}|\sqrt{2}\alpha,0\rangle_{9,10}|0,\sqrt{2}\alpha\rangle_{11,12}|\alpha,\alpha,-\alpha\rangle_{346} + \\ &|0,\sqrt{2}\alpha\rangle_{7,8}|0,\sqrt{2}\alpha\rangle_{9,10}|\sqrt{2}\alpha,0\rangle_{11,12}|-\alpha,-\alpha,\alpha\rangle_{346} + \\ &|0,\sqrt{2}\alpha\rangle_{7,8}|0,\sqrt{2}\alpha\rangle_{9,10}|0,\sqrt{2}\alpha\rangle_{11,12}|-\alpha,-\alpha,-\alpha\rangle_{346}\big) + \\ &a_1b_0\big(|0,-\sqrt{2}\alpha\rangle_{7,8}|0,-\sqrt{2}\alpha\rangle_{9,10}|\sqrt{2}\alpha,0\rangle_{11,12}|\alpha,\alpha,\alpha\rangle_{346} + \\ &|0,-\sqrt{2}\alpha\rangle_{7,8}|0,-\sqrt{2}\alpha\rangle_{9,10}|0,\sqrt{2}\alpha\rangle_{11,12}|\alpha,\alpha,-\alpha\rangle_{346} + \\ &|-\sqrt{2}\alpha,0\rangle_{7,8}|-\sqrt{2}\alpha,0\rangle_{9,10}|\sqrt{2}\alpha,0\rangle_{11,12}|-\alpha,-\alpha,\alpha\rangle_{346} + \\ &|-\sqrt{2}\alpha,0\rangle_{7,8}|-\sqrt{2}\alpha,0\rangle_{9,10}|0,\sqrt{2}\alpha\rangle_{11,12}|-\alpha,-\alpha,-\alpha\rangle_{346}\big) + \\ &a_0b_1\big(|\sqrt{2}\alpha,0\rangle_{7,8}|\sqrt{2}\alpha,0\rangle_{9,10}|0,-\sqrt{2}\alpha\rangle_{11,12}|\alpha,\alpha,\alpha\rangle_{346} + \\ &|\sqrt{2}\alpha,0\rangle_{7,8}|\sqrt{2}\alpha,0\rangle_{9,10}|-\sqrt{2}\alpha,0\rangle_{11,12}|\alpha,\alpha,-\alpha\rangle_{346} + \\ &|0,\sqrt{2}\alpha\rangle_{7,8}|0,\sqrt{2}\alpha\rangle_{9,10}|0,-\sqrt{2}\alpha\rangle_{11,12}|-\alpha,-\alpha,\alpha\rangle_{346} + \\ &|0,\sqrt{2}\alpha\rangle_{7,8}|0,\sqrt{2}\alpha\rangle_{9,10}|-\sqrt{2}\alpha,0\rangle_{11,12}|-\alpha,-\alpha,-\alpha\rangle_{346}\big) + \\ &a_1b_1\big(|0,-\sqrt{2}\alpha\rangle_{7,8}|0,-\sqrt{2}\alpha\rangle_{9,10}|0,-\sqrt{2}\alpha\rangle_{11,12}|\alpha,\alpha,\alpha\rangle_{346} + \\ &|0,-\sqrt{2}\alpha\rangle_{7,8}|0,-\sqrt{2}\alpha\rangle_{9,10}|-\sqrt{2}\alpha,0\rangle_{11,12}|\alpha,\alpha,-\alpha\rangle_{346} + \\ &|-\sqrt{2}\alpha,0\rangle_{7,8}|-\sqrt{2}\alpha,0\rangle_{9,10}|0,-\sqrt{2}\alpha\rangle_{11,12}|-\alpha,-\alpha,\alpha\rangle_{346} + \\ &|-\sqrt{2}\alpha,0\rangle_{7,8}|-\sqrt{2}\alpha,0\rangle_{9,10}|-\sqrt{2}\alpha,0\rangle_{11,12}|-\alpha,-\alpha,-\alpha\rangle_{346} \end{aligned} \qquad (6)$$

which gives, after an involved simple manipulation,

$$|\phi\rangle_{7,8,9,10,11,12,3,4,6} \propto [(a_0b_0|\sqrt{2}\alpha,0\rangle_{7,8}|\sqrt{2}\alpha,0\rangle_{9,10}$$

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$$a_1b_0|0,-\sqrt{2\alpha}\rangle_{7,8}|0,-\sqrt{2\alpha}\rangle_{9,10})|\alpha,\alpha\rangle_{3,4} + (a_0b_0|0,\sqrt{2\alpha}\rangle_{7,8}|0,\sqrt{2\alpha}\rangle_{9,10} + a_1b_0|-\sqrt{2\alpha},0\rangle_{7,8}|-\sqrt{2\alpha},0\rangle_{9,10})|-\alpha,-\alpha\rangle_{3,4}] \otimes(|\sqrt{2\alpha},0\rangle_{11,12}|\alpha\rangle_6 + |0,\sqrt{2\alpha}\rangle_{11,12}|-\alpha\rangle_6) + [(a_0b_1|\sqrt{2\alpha},0\rangle_{7,8}|\sqrt{2\alpha},0\rangle_{9,10} + a_1b_1|0,-\sqrt{2\alpha}\rangle_{7,8}|0,-\sqrt{2\alpha}\rangle_{9,10})|\alpha,\alpha\rangle_{3,4} + (a_0b_1|0,\sqrt{2\alpha}\rangle_{7,8}|0,\sqrt{2\alpha}\rangle_{9,10} + a_1b_1|-\sqrt{2\alpha},0\rangle_{7,8}|-\sqrt{2\alpha},0\rangle_{9,10}) + |-\alpha,-\alpha\rangle_{3,4}] \otimes(|0,-\sqrt{2\alpha}\rangle_{11,12}|\alpha\rangle_6 + |-\sqrt{2\alpha},0\rangle_{11,12}|-\alpha\rangle_6) \quad (7)$$

Step2- Alice performs photon-count measurements by the photon-number resolving detectors $D_{7,8}$, for output modes (7,8) from BPS-1 getting (p,q) number of photons and $D_{9,10}$, for output modes (9,10) from BPS-2 getting (p', q') number of photons. Similarly, Bob does the same measurements via detectors $D_{11,12}$ for output modes (11,12) from BPS-3 getting (p'', q'') number of photons.

That is to say, (6) collapses to provide the heralded state vector, $|\phi(p,q,p',q',p'',q'')\rangle_{3,4,6}$ where,

$$|\phi(p,q,p',q',p'',q'')\rangle_{3,4,6} = {}_7\langle p|_8\langle q|_9\langle p'|_{10}\langle q'|_{11}\langle p''|_{12}\langle q''|\phi\rangle_{7,8,9,10,11,12,3,4,6}, \quad (8)$$

Now, Alice sends the results of photon-number measurements, p, p', q and q' to Bob through classical channel and, at the same time, Bob communicates his photon-numbers p'', q'' to Alice via classical channel too.

Step 3-Alice may find photons $p \neq 0, q = 0$ from detectors $D_{7,8}$, photons $p' \neq 0, q' = 0$, from detectors $D_{9,10}$ in Alice's lab and photons $p'' \neq 0$, $q'' = 0$ may be detected in Bob's lab from detectors $D_{10,11}$. It is, therefore, (8) yields the heralded state-vector

$$|\phi'\rangle_{2,3,6} \propto a_0b_0|\alpha,\alpha,\alpha\rangle_{3,4,6} + (-1)^{p''}a_0b_1|\alpha,\alpha,-\alpha\rangle_{3,4,6} + (-1)^{p+p'}a_1b_0|-\alpha,-\alpha,\alpha\rangle_{3,4,6} + (-1)^{p+p'+p''}a_1b_1|-\alpha,-\alpha,-\alpha\rangle_{3,4,6} \quad (8a)$$

upto a normalization constant. For the other possibilities $(p,p',p''=0)$ and $(q,\ q',q'' \neq 0)$ the heralded state-vector may be read as

$$|\phi''\rangle_{2,3,6} \propto a_0b_0|-\alpha,-\alpha,-\alpha\rangle_{3,4,6} + (-1)^{q''}a_0b_1|-\alpha,-\alpha,\alpha\rangle_{3,4,6} + a_1b_0(-1)^{q+q'}|\alpha,\alpha,-\alpha\rangle_{3,4,6} + (-1)^{q+q'+q''}a_1b_1|\alpha,\alpha,\alpha\rangle_{3,4,6} \quad (8b)$$

In deriving (7a) and (7b), expressions $\langle m|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2}\frac{\alpha^m}{\sqrt{m!}}$ and $\langle m|-\alpha\rangle = (-1)^m e^{-\frac{1}{2}|\alpha|^2}\frac{\alpha^m}{\sqrt{m!}}$ have been used, m being non-zero integer.

Step 4- According to (8a) and (8b), the different possibilities of number of photons p,p',p'' and q,q',q'' from detectors $D_{7,8,9,10,11,12}$ and their consequent 'heralded states' vectors along with local unitary operations at Alice's and Bob's labs may be listed in given Table-I.

Thus, the ABQT protocol is successfully realized by classical communications of number of photons $(p,q),(p',q')\ and\ (p'',q'')$ and local unitary operations at Alice's and Bob's labs,

$$(a_0|\alpha,\alpha\rangle + a_1|-\alpha,-\alpha\rangle)_{3,4} \otimes (b_0|\alpha\rangle + b_1|-\alpha\rangle)_6 \quad (9)$$

upto a normalization constant.

III. Probability of Success and Fidelity

Finally, the probability of finding the number of photons p,p',p'' and the number of photons q,q',q'' in modes (7,8,9,10) and (11,12) may be derived by

$$P(p,p',p'',q,q',q'') = |{}_7\langle p|_9\langle p'|_{11}\langle p'|_8\langle q|_{10}\langle q'|_{12}\langle q''|\phi\rangle_{7,8,9,10,11,12,3,4,6}|^2 \quad (10)$$

After a simple calculation, (10) gives the success probabilities $P(0,0,0,p,p',p'')$ and $P(p,p',p'',0,0,0)$ for even p,p',p'' to be,

$$P_{even} = \sum_{even} P(0,0,0,p,p',p'') + \sum_{even} P(p,p',p'',0,0,0) = \frac{1}{2}, \quad (11)$$

for $|\alpha| \to \infty$, signified physically by either of the two detection-events occurring exemplified by Table-I and II.

It may be seen from (11) that the success probability is independent of α, the amplitude of optical coherent field and the unknown probability amplitudes a_0, a_1 of Alice information-state and those of b_0, b_1 of Bob's information-state.

In order to analyze the performance of quantum teleportation one may evaluate fidelity, i.e, the overlapping of 'to be teleported' information-state and the states obtained after ABQT gets completed. Since the asymmetric bidirectional quantum teleported state, (9) are same as the information-states possessed by Alice and Bob, fidelity is evaluated to have nearly unit value, if $|\alpha| \to \infty$, i.e, perfect ABQT.

Since the ABQT-scheme worked out, here, utilizes experimentally available optical coherent states, multi-modal entangled coherent states and simple optical elements such as beam splitters, phase shifters and elementary unitary (gate) operation, the ABQT-protocol could also be realized experimentally.

References

[1] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, "Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels," *Phys. Rev. Lett.*, vol. 70, no. 13, pp. 1895–1899, 1993, doi: 10.1103/PhysRevLett.70.1895.

[2] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, "Experimental quantum teleportation," *Nature*, vol. 390, no. 6660, pp. 575–579, 1997, doi: 10.1038/37539.

[3] X.-M. Hu, Y. Guo, B.-H. Liu, C.-F. Li, and G.-C. Guo, "Progress in quantum teleportation," *Nat. Rev. Phys.*, vol. 5, no. 6, pp. 339–353, 2023, doi: 10.1038/s42254-023-00588-x.

[4] S. L. Braunstein and H. J. Kimble, "Teleportation of Continuous Quantum Variables," *Phys. Rev. Lett.*, vol. 80, no. 4, pp. 869–872, Jan. 1998, doi: 10.1103/PhysRevLett.80.869.

[5] A. Furusawa, J. L. Sørensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, "Unconditional quantum teleportation," *Science (80-.).*, vol. 282, no. 5389, pp. 706–709, Oct. 1998, doi: 10.1126/science.282.5389.706.

[6] P. T. Cochrane, G. J. Milburn, and W. J. Munro, "Macroscopically distinct quantum-superposition states as a bosonic code for amplitude damping," *Phys. Rev. A*, vol. 59, no. 4, pp. 2631–2634, Apr. 1999, doi: 10.1103/PhysRevA.59.2631.

[7] V. V. Dodonov, I. A. Malkin, and V. I. Man'ko, "Even and odd coherent states and excitations of a singular oscillator," *Physica*, vol. 72, no. 3, pp. 597–615, Mar. 1974, doi: 10.1016/0031-8914(74)90215-8.

[8] T. C. Ralph, A. Gilchrist, G. J. Milburn, W. J. Munro, and S. Glancy,

"Quantum computation with optical coherent states," *Phys. Rev. A*, vol. 68, no. 4, p. 42319, Oct. 2003, doi: 10.1103/PhysRevA.68.042319.
[9] S. J. Van Enk and O. Hirota, "Entangled coherent states: Teleportation and decoherence," *Phys. Rev. A. At. Mol. Opt. Phys.*, vol. 64, no. 2, pp. 1–6, 2001, doi: 10.1103/PhysRevA.64.022313.
[10] X. Wang, "Quantum teleportation of entangled coherent states," *Phys. Rev. A*, vol. 64, no. 2, p. 22302, Jul. 2001, doi: 10.1103/PhysRevA.64.022302.
[11] B. C. Sanders, "Entangled coherent states," *Phys. Rev. A*, vol. 45, no. 9, p. 6811, May 1992, doi: 10.1103/PhysRevA.45.6811.
[12] H. Prakash, N. Chandra, R. Prakash, and Shivani, "Improving the teleportation of entangled coherent states," *Phys. Rev. A - At. Mol. Opt. Phys.*, vol. 75, no. 4, p. 044305, Apr. 2007, doi: 10.1103/Phys. Rev. A.75.0 44305.
[13] A. Karlsson and M. Bourennane, "Quantum teleportation using three-particle entanglement," *Phys. Rev. A*, vol. 58, no. 6, p. 4394, Dec. 1998, doi: 10.1103/PhysRevA.58.4394.
[14] C. P. Yang, S. I. Chu, and S. Han, "Efficient many-party controlled teleportation of multiqubit quantum information via entanglement," *Phys. Rev. A*, vol. 70, no. 2, p. 022329, Aug. 2004, doi: 10.1103/PhysRevA.70.022329.
[15] L. Dong, X. M. Xiu, Y. J. Gao, Y. P. Ren, and H. W. Liu, "Controlled three-party communication using GHZ-like state and imperfect Bell-state measurement," *Opt. Commun.*, vol. 284, no. 3, pp. 905–908, Feb. 2011, doi: 10.1016/J.OPTCOM.2010.09.089.
[16] S. Hassanpour and M. Houshmand, "Efficient controlled quantum secure direct communication based on GHZ-like states," *Quantum Inf. Process.*, vol. 14, no. 2, pp. 739–753, 2015, doi: 10.1007/s11128-014-0866-z.
[17] S. Gangopadhyay, T. Wang, A. Mashatan, and S. Ghose, "Controlled quantum teleportation in the presence of an adversary," *Phys. Rev. A*, vol. 106, no. 5, p. 52433, Nov. 2022, doi: 10.1103/PhysRevA.106.052433.
[18] R. K. Pandey, R. Prakash, and H. Prakash, "Controlled Quantum Teleportation of Superposed Coherent State Using GHZ Entangled Coherent State," *Int. J. Theor. Phys.*, vol. 58, no. 10, pp. 3342–3351, 2019, doi: 10.1007/s10773-019-04207-3.
[19] S. F. Huelga, J. A. Vaccaro, A. Chefles, and M. B. Plenio, "Quantum remote control: Teleportation of unitary operations," *Phys. Rev. A*, vol. 63, no. 4, p. 42303, Mar. 2001, doi: 10.1103/PhysRevA.63.042303.
[20] X.-W. Zha, Z.-C. Zou, J.-X. Qi, and H.-Y. Song, "Bidirectional Quantum Controlled Teleportation via Five-Qubit Cluster State," *Int. J. Theor. Phys.*, vol. 52, no. 6, pp. 1740–1744, 2013, doi: 10.1007/s10773-012-1208-5.
[21] C. Shukla, A. Banerjee, and A. Pathak, "Bidirectional Controlled Teleportation by Using 5-Qubit States: A Generalized View," *Int. J. Theor. Phys.*, vol. 52, no. 10, pp. 3790–3796, 2013, doi: 10.1007/s10773-013-1684-2.
[22] S. Hassanpour and M. Houshmand, "Bidirectional quantum teleportation via entanglement swapping," in *2015 23rd Iranian Conference on Electrical Engineering*, 2015, pp. 501–503. doi: 10.1109/IranianCEE.2015.7146267.
[23] V. Verma, "Bidirectional quantum teleportation and cyclic quantum teleportation of multi-qubit entangled states via G-state," *Int. J. Mod. Phys. B*, vol. 34, no. 28, p. 2050261, Oct. 2020, doi: 10.1142/S0217979220502616.
[24] V. Verma, "Bidirectional Quantum Teleportation by Using Two GHZ-States as the Quantum Channel," *IEEE Commun. Lett.*, vol. 25, no. 3, pp. 936–939, 2021, doi: 10.1109/LCOMM.2020.3036587.
[25] Y. Mafi, P. Kazemikhah, A. Ahmadkhaniha, H. Aghababa, and M. Kolahdouz, "Bidirectional quantum teleportation of an arbitrary number of qubits over a noisy quantum system using 2 n Bell states as quantum channel," *Opt. Quantum Electron.*, vol. 54, no. 9, pp. 1–13, Sep. 2022, doi: 10.1007/s11082-022-03951-x.
[26] V. Verma, N. Singh, and R. S. Singh, "Improvement on Quantum Teleportation of Three and Four Qubit States Using Multi-Qubit Cluster States," *Int. J. Theor. Phys.*, vol. 60, no. 10, pp. 3973–3981, 2021, doi: 10.1007/s10773-021-04949-z.
[27] B. S. Choudhury and S. Samanta, "A controlled protocol for asymmetric cyclic quantum state transfer between three parties," *Phys. Scr.*, vol. 95, no. 1, p. 015101, Nov. 2019, doi: 10.1088/1402-4896/ab3d43.
[28] M. huang Sang, "Bidirectional Quantum Teleportation by Using Five-qubit Cluster State," *Int. J. Theor. Phys.*, vol. 55, no. 3, pp. 1333–1335, Mar. 2016, doi: 10.1007/s10773-015-2774-0.
[29] X. Yang, D. Li, J. Zhou, Y. Tan, Y. Zheng, and X. Liu, "Research on Key Technologies of Controlled Bidirectional Quantum Teleportation," *Int. J. Theor. Phys.*, vol. 62, no. 4, pp. 1–14, Apr. 2023, doi: 10.1007/s10773-022-05256-x.
[30] Y.-X. Chen, J. Du, S.-Y. Liu, and X.-H. Wang, "Cyclic quantum teleportation," *Quantum Inf. Process.*, vol. 16, no. 8, p. 201, 2017, doi: 10.1007/s11128-017-1648-1.
[31] V. Verma, "Cyclic quantum teleportation via GHZ-like state," *Mod. Phys. Lett. A*, vol. 35, no. 40, p. 2050333, Nov. 2020, doi: 10.1142/S0217732320503332.
[32] R.-J. Liu, M.-Q. Bai, F. Wu, and Y.-C. Zhang, "Cyclic-controlled quantum operation teleportation in noisy environment," *Int. J. Quantum Inf.*, vol. 17, no. 07, p. 1950052, Oct. 2019, doi: 10.1142/S0219749919500527.
[33] J. Gu, T. Hwang, and C.-W. Tsai, "On the Controlled Cyclic Quantum Teleportation of an Arbitrary Two-Qubit Entangled State by Using a Ten-Qubit Entangled State," *Int. J. Theor. Phys.*, vol. 59, no. 1, pp. 200–205, 2020, doi: 10.1007/s10773-019-04311-4.
[34] J. Shi, X. Zhang, and Y. Zhu, "Cyclic Controlled Quantum Teleportation Using Three-Dimensional Hyper-Entangled State," *Int. J. Theor. Phys.*, vol. 58, no. 9, pp. 3036–3048, 2019, doi: 10.1007/s10773-019-04183-8.
[35] V. Verma and R. S. Singh, "Simultaneous Quantum Teleportation within a Quantum Network," *Int. J. Theor. Phys.*, vol. 61, no. 7, p. 191, 2022, doi: 10.1007/s10773-022-05177-9.
[36] N. Singh, N. Singh, A. Pathak, V. Verma, and R. S. Singh, "Multi-directional controlled quantum teleportation by using nine-qubit entangled state as a single quantum channel," *Opt. Quantum Electron.*, vol. 55, no. 10, p. 932, 2023, doi: 10.1007/s11082-023-05198-6.
[37] R. K. Pandey, P. S. Yadav, R. Prakash, and H. Prakash, "Controlled Bidirectional Quantum Teleportation of Superposed Coherent State Using Five-mode Cluster-type Entangled Coherent State as a Resource," *Int. J. Theor. Phys.*, vol. 61, no. 4, p. 104, 2022, doi: 10.1007/s10773-022-05080-3.
[38] S. Aliloute, A. El Allati, and I. El Aouadi, "Bidirectional teleportation using coherent states," *Quantum Inf. Process.*, vol. 20, no. 1, p. 29, 2021, doi: 10.1007/s11128-020-02945-6.
[39] N. Ikken, A. Slaoui, R. Ahl Laamara, and L. Btissam Drissi, "Bidirectional quantum teleportation of even and odd coherent states through the multipartite Glauber coherent state: Theory and implementation," *arXiv e-prints*. pp. 1–14, Jun. 01, 2023. doi: 10.48550/arXiv.2306.00505.
[40] E. O. Kiktenko, A. A. Popov, and A. K. Fedorov, "Bidirectional imperfect quantum teleportation with a single Bell state," *Phys. Rev. A*, vol. 93, no. 6, p. 62305, Jun. 2016, doi: 10.1103/PhysRevA.93.062305.