

An outlined tour of geometry and topology as perceived through physics and mathematics emphasizing geometrization, elliptization, uniformization, and projectivization for Thurston's 8-geometries covering Riemann over Teichmüller spaces

Deep Bhattacharjee¹

Equivalence, duality, and invariance are the pin-points of unification in modern theoretical physics that got the twist of topologies when going beyond the notions of differential and conformal domains to geometries to the symplectic norm of topologies with the pillars being the algebraic geometry taking the counting of specified states through the Enumerative ones.

Terrell – Penrose; Gauss-Bonnet; Green's Function; Poincaré' and Perelman; Teichmüller Space, Ricci Flow and Surgery, Sasaki – Einstein

Structures of nature can be represented through geometry where the more complex – hypercomplex – hyperbolic complex – compact – smooth – orientable with several varieties have been identified, classified, analyzed so on. Formulating the typical structures with or without singularities there's a norm for canonical or relativistic ones. The Gauss – Bolyai – Lobachevsky while takes the control of hyperbolic structures, so as, the Riemann, considering the positive geometry of relativity makes the bulge outward in a more elliptical way. If there is indeed the topology satisfying the hyperbolic notions or saddle geometries then there would be Tachyons flowing fast rolling down through slopes violating every norm of the negatively curved space by a positively curved ball. Others that are there yet to be proven, already have been axioms through rigorous classifications yielding results in the beauty of Poincaré, Thurston, Mostow, Gauss, Einstein, Perelman, and more to the inverse starting all from Euclid – the coordinates being Cartesian rather than Curvilinear. Projections being projective shows us the illusion of two rail tracks meeting at a point in a distance. Because the geometry of the Earth's surface is elliptic having the omega and Kretschmann greater than zero – still the locally defined place is the Euclidean having the only constraint is the 'sky' as the reference or the 'distance' to where the tracks projects being singular. Thurston proved that any geometric three-dimensional surface can be analyzed by 3 distinct geometries – Elliptic Hyperbolic, and Euclidean but also classified through 'point stabilizers' in 8 – groups. Proceeding further some spaces have frequencies through the geometry and related topology. If there is Riemann to provide the relations between sides and angles, then there's a Conformal for making the angular mapping equal over other projections in symmetries. Else, there is symplectic to pursue the area-volume relation taking the Enumerative for the counting of the specified states as happened in the embedding structures or manifolds. Difficulty to pursue or observe the structures of the natures to an approx of a division of 1 part from 36,000 crores in the tiniest of the domains taking through the horizons or the 94 light-years wide structures of the universes where even there can be geometries 'if not a void' in between two universes in the multiverse chains. If symplectic topology performs the operations over manifolds by examining its structures then the algebraic geometry proves equipment or tolls to him for making his performance faster. From the hyperkähler to the Kähler to the compact Kähler or Calabi-Yau taking over K-3, Kummer, de Rham cohomology for some specific holomorphic bundles there are creative notions in stable/semi-stable/unstable forms for all conjectures of Strominger–Yau –Zaslow, Calabi–Yau, Henri Poincaré, geometrization, projectivization with the separation of Euclidean to hyperbolic 3-folds via Dehn surgery – the inaccessible world of topology is still mapping from domains to co-domains

¹itsdeep@live.com

making the connectivity of Gromov's and Witten's, the cohomology of Floer, etale, where there are dualities and equivalence, rings and commutations. Then comes the FANO with the Picard – Lebanese forms over complex Jacobians. The duality of Poincare mostly being the center of the Nomikov ring, de Rahm cohomology, and Gromov connections, there are also the rings of Goreshteins over Cohen – Macaulay degrees all through the Bogomolov–Tait–Todorov theorem taking the structures that are most fundamental being the origin of particles from strings itself through permutations and cycles of permutations sometimes being complex, sometimes for higher degree polynomials there's the combinatorics. There the representations, categories, and symplectic classes such that high order generalizations for the projective space can be made over complex topologies being observable as the signature criterion in understanding the workings of the mother nature^[1-3].

Apart from other mechanics if one considers the playground of Newton or the Newtonian geometry then it won't make any proper sense as his theory of gravity is independent of geometry. Although not in all – as gravity originates from mass, then that mass happens to take a shape which is a structure or a manifold in any counting dimensions but two things can't correlate with his views;

- [1] Association on the effects of curvatures and masses with the geometries related to the attractive force of gravity.
- [2] The complex structures with other relatable topologies that generates mass with being transformed from Planck Brane to TeV Brane.

Most of these notions are generalized in the differential forms of geometry where there are domains and co-domains with bijections, surjections injects are taking place giving distinct classes of mapping being preserved through topologies. The homological, cohomological, quantum cohomological manifolds with induced isomorphisms taking over moduli spaces can in terms denote several notions of connectivity that are significant in the field theories of strings, and algebras, homeomorphisms, rings. The distinction among structures having genus or not is so much prominent that any sort of homeomorphisms is prevented on account of Euler characteristics of polyhedron equations with some added ' $-2g$ ' on RHS^[4].

The most compact structures that are being fundamental having genus = 3 appears in the topological manifolds concerning strings where the field theories of Witten's are there making the three most important contributions as ghost numbers,

- [1] Associativity
- [2] Cyclicity
- [3] BRST Invariance

Realizing the genre of strings are the dualities being T or S or both T-S can be achieved over several field theories where the two most important being performed over reconciled twisted K – theories or others in the stacks of Deligne–Mumford being generalized over^[5,6],

- [1] Gromov – Witten invariants
- [2] Atiyah – Hirzebruch spectral sequences

Every notion of these theories is related to theoretical physics by the amalgamation between the quantum world to the relativistic world making the beautiful transformations, to say just five^[3],

- [1] Nambu – Goto makes the classical action relativistic.
- [2] Klein – Gordon incorporated the special theory of relativity.
- [3] Heisenberg shows in another perspective to increase the length of the particle into strings to prevent catastrophes.
- [4] Beyond the standard model and supersymmetry
- [5] Invariance takes place with the increase of dimensions.

Almost everything has its roots in Riemann and the curvature tensors. The simple world of Euclid is difficult to force through the topologies where there are curvatures in every geometry with a few slices being Euclidean that also if that structure is solid, else locally defined. The connection coefficients, covariant derivatives, and parallel transports from the angular difference between initial and final vectors are taken in every equation of Einstein^[7].

Hilbert on the other hand has a space as Hilbert space for the quantum field theory to iterate or play on being devoid of any curvatures hence gravity. But the fulfillment of physics can only come if the field of Hilbert can be coped with the field of Riemann where the quantum particles are playing over curved surfaces characterized by geometries and complex topologies with gravity being the sole keeper of this beautiful playground. This can be observed in the form of gamma – integrals with the momentum limiting to infinity through Einstein – Hilbert actions where from onwards loop (2) the divergences got so prominent that the renormalization has been a tool to withstand the mathematics or the equations from reaching out to Mars. Other modifications being done over Palatini actions with Ashtekar variables, Wilson loops and Triads, localization principles of supersymmetry, Regge poles and scattering matrices, the harmonic oscillator giving rise to left-right string orientation producing graviton – dilaton. Renormalization being a necessity for calculations on divergences yielding from quantum gravity, the features tools are fixed point flows, and asymptotic safety. Others are there through Regge calculus and causal dynamic triangulation^[7,8].

If Einstein wouldn't realize the geometric nature of gravity then three things would be unknown to humans,

- [1] Gravity as a mere force
- [2] The geometry is hyperbolic with the elliptic stars sitting over them making the planets with their natural satellites revolve around that star by the generation of angular momentum from that curvatures.
- [3] The interesting world of hyperbolic stars sitting in space makes it hyperbolic with the movements taking far more than that countable in theories.

If the origin of black holes is from Schwarzschild with just a coordinate singularity having no spin, then the Kerr form in Boyer–Lindquist coordinates, the Eddington–Finkelstein coordinates where there's spin being the realistic description of today's black holes being compact with the light rays escaping from the Cauchy – Horizon in future. The ring-type singularity and to get through them, the space-time warpage with the tilting of light cones in spacelike trajectories with the more abstract form of non-BPS black holes there's a lot to explore with charges getting counteracted with gravity^[9,10].

The supersymmetry gives a prediction about the nature of cosmological constant and the cancellations between the positive-negative all though the statistics of^[11],

- [1] Maxwell – Boltzmann: $1/e^{(E/KT)}$ Where "e" is Exponential, "K" is Boltzmann Constant, "E" is Energy, "T" is Temperature.
- [2] Fermi – Dirac: $1/e^{(E - EF/KT + 1)}$ Where "EF" is the Valence Band of the Electrons. This is also known as the Fermi Sea or the level below which all the electron states due to energy constraints. It obeys Pauli Exclusion Principle.
- [3] Bose – Einstein: $1/e^{(E/KT - 1)}$ where the '-1' term is simply the difference in behavior from the Maxwell – Boltzmann. This is an important state which was predicted by Albert Einstein and Satyendranath Bose and the amazing property rises when the "T" term goes 0 Kelvin then the whole state will become zero which means all the electrons occupy the ground states and they collectively behave as a super-atom called as Bose-Einstein Condensate. This does not obey the Pauli exclusion principle. To the takeovers of Pauli through his exclusion principles having,

- Spin quantum number: which can be either -1/2 or +1/2 represents an up spin and down spin.

- Spin quantum number: which can be either $-1/2$ or $+1/2$. Represents a up spin and down spin.
- Magnetic moment quantum number: which denotes Simply $-L, 0, +L$ & if L is 0, Then We can get the 'S' Orbitals, if $N=1$, We can get the 'P' Orbitals as it corresponds to $-1, 0, +1$. If L is 2 we can get 'D' orbitals which are $-2, -1, 0, +1, +2$. If L is 3, We get 'F' Orbitals that is $-3, -2, -1, 0, +1, +2, +3$. "S" orbital has 1 STATES, "P" orbital has 3 STATES, "D" orbital has 5 STATES, "F" orbital has 7 States.
- Angular quantum number: which deals with (L) as $0, 1, 2, 3, \dots, (N-1)$ The energy State of the Orbital.
- Principle quantum number: Which Shows the Level of Orbitals like $N=1, 2, 3, 4, \dots$ with each level the amount of Electrons is simply given by $2(N)^2$ where for $N=1$, we have 2 Electrons, For $N=2$, we have 8 electrons, For $N=3$, we have 18 Electrons and so on.

Calabi – Yau structures being governed by the supersymmetric natures of gravity or the supergravity, in essence, give rise to producing several variables for 'N' thus making each CY take different shapes depending upon that 'N'. there are CY with terminal singularities, some being non-singular with the much higher dimensional projective varieties described via complete intersections, morphisms, commutative algebras, categories, representation theories, super Lie algebras with orthosymplectic groups obeying Poincare–Birkhoff–Witt Theorems^[12].

Association of Einstein through 3 different metrics gives topology a new roadway to explore^[1],

- [1] Einstein – Kähler: The metric of the complex manifold in a Riemann form being both of incorporating the significant Ricci flat Kähler i.e. Calabi –Yau.
- [2] Einstein – Hermite: A 2 –form connection as associated with Kähler having the Yang–Mills formulations.
- [3] Sasaki – Einstein: A Riemann metric for a contact surface defining the Riemann cone.

When the metrics of Einstein got amalgamated with the equations of Dirac, Pauli, Fermi, Planck, Bohr, Schrodinger, Feynman then the outcome is a supernova with^[13,14],

- [1] The spooky world of entanglement has Bell's inequality for Einstein – Podolsky – Rosen Paradox.
- [2] The fantastic world of Einstein – Rosen Bridges that although are not traversal but can be made traversal fully or to some extent by incorporating the brilliant works of,
 - Throne and Morris
 - Raychaudhuri Equations and the modified dynamics of general relativity appeared in $f(R, T)$ theories.

[3] The tiny world of strings having transformed from Bosonic to Bosonic – Fermionic incorporating supersymmetry with the addition of 11-dimensional supergravity giving M – Theory of Witten were among the 1+5+1 theories^[3],

1. Bosonic
2. Type – I
3. Type – II(A)
4. Type – II(B)
5. Heterotic $SO(32)$
6. Heterotic $E_8 \times E_8$
7. 11-dimensional supersymmetric gravity or SUGRA.
 - Where Type – II (B) is the most interesting with both T and S duality giving birth to F – Theory with additional time dimensions all encountering the elliptic cycles, and gravities in a new way.

This is not to be forgotten that when there's special relativity and the speed of the light then Einstein always comes with Maxwell in one way or other as the most interesting thing is the "permeability of free space" that is " $\mu^0.\epsilon$ " which has a value of $1/c^2$ from which Einstein borrowed the formulae and used it in relativity^[15].

- Maxwell equations are there in the forms of – Gauss Electricity law, Gauss magnetism law, Ampere law, and Faraday's law in both integral and vector forms,
 - [1] The first law states the divergence vector of an electric field is the flux density of the current flowing.
 - [2] The second law states the divergence of the magnetic field which is always zero as the magnet has both the positive and negative poles.
 - [3] The third law states the current when induced through a coil produced a magnetic field and thereby uses a curl vector for that.
 - [4] The fourth law unites electricity with magnetism and gives the constant value of the speed of the light along with a displacement current correction in curl vector forms.

Getting back to the implicit notions of relativity while coming back to Einstein – Relativity has the property of describing the mechanics of black hole cosmology and thus there exists a related definition of geometry with 3 curvatures^[16],

- [1] A maximally symmetric manifold being Lorentzian gives the Anti-de Sitter space being generalized in any dimensions taking the dimension of time as 1. This satisfies the hyperbolic geometry or a geometry having a constant negative scalar curvature with the de Sitter being the space of positive curvatures.
- [2] Relativity doesn't distinguish between space and time – thus making it united under the common perception of space-time. This denotes a special type of space of zero curvature called the Minkowski space where one can see the Euclidean line element taking the shape of a metric tensor that also is a scalar having 2 types of signatures $(+, -, -, -)$ and $(-, -, -, -)$ enabling us to know the particle trajectories that are being identified as,
 - Timelike – Less than the speed of light.
 - Lightlike – Equal to the speed of light.
 - Spacelike – Faster than the speed of light.

- [3] AdS comes with correspondence for gauge – gravity relation called the AdS/CFT correspondence making the gateway of electromagnetic, strong, and weak forces. CFT is the conformal geometry being corresponded in AdS spaces.

Early days before the origin of the theory of cyclic cosmology FLRW or FRW metrics have shown us the possible fate of the universes where there can be possibilities of – Big Rip – Big Crunch – Exponential Expansions. However, the Ekpyrotic or cyclic cosmological model gives us the account of p-Brane or Polchinsky Brane or D(p)-Brane where D stands for Dirichlet. Being the boundary conditions over the Branes where the standard model plays its role – the boundaries are there for the strings but being categorized under two factors^[3],

- [1] Open strings being attached to boundaries are restricted for interdimensional travel and are photons.
- [2] Closed strings being no endpoints on Branes can move in the higher dimensional bulks and the example of such is a graviton.

String theory as admits supersymmetry thus every Boson has a corresponding fermion while every fermion has a corresponding boson,

- For bosons photon, there is fermion photino.
- For fermions electron, there is boson selectron.

Higher-order dimensions are for more degrees of freedom where the topologies of warping or curling are patched up in a small area making the tiny structures existing in the space-time grids are the Calabi – Yau manifolds admitting supersymmetry. The first higher degree generalization to 5 was made by Kaluza and Klein by extending Einstein's general relativity from 4 to 5 dimensions where the momentum of winding gets to charges making the notion that perceiving structures in distance can be seen with 1 dimension with others being vanished. Relativistic solutions got modified and the effects resulting from the theory got more concrete – to say^[16,17],

- Fitz – Gerald Lorentz contraction taking place in any object approaching the speed of light is replaced by rotations over Terrell - Penrose effects.

Any simply connected Riemann surface is conformally equivalent to any universal covering over discrete groups being a simply connected domain^[18],

- [1] The unit disc is a genus 1 hyperbolic surface.
- [2] The complex plane is a genus 1 Riemann surface.
- [3] The Riemann sphere being the genus 0 Riemann manifolds.

Every Riemann surface having simple connectivity is biholomorphic to each one of the above (3) for the bijective holomorphism from every non-empty open subset into the unit disc. For the universal cover, any Riemann surface can be split into 3 categories^[19],

- [1] Elliptic as the Riemann sphere for +1 curvatures.
- [2] Parabolic is the universal cover of 0 curvatures.
- [3] Hyperbolic having the covering associated to -1 curvatures.

The Riemann with the uniformization classifiers the Euler's characteristics for genus g as $2 - 2g$ where for every hyperbolic 2-manifolds criterion one can have the Gauss – Bonnet theorem covering a complete justification over the area element $d\omega$ taking the 2-dimensional compact manifold m for the Gaussian curvature \mathcal{G} for the boundaries ∂m noting Euler characteristics for the functional manifold $\mathcal{X}(m)$ gives for the related geodesic Γ_g ^[20],

$$\int_m \mathcal{G} d\omega = 2\pi\mathcal{X}(m) - \int_{\partial m} \Gamma_g d\omega ds$$

For the constant curvatures of these isometry groups, the harmonic function and Green's function take place invoking the non-linear differentials for conformal metrics. Thus, there exist 3-categories for classifications^[21],

- [1] The 2 – sphere having \mathcal{X} equals 2.
- [2] The Euclidean having \mathcal{X} equals 0.
- [3] The hyperbolic satisfies the above mentioned Gauss – Bonnet for any negative \mathcal{X} .

For any Riemann moduli, the Teichmuller space exists as the covering orbifold for marked homeomorphisms on the complex topological manifold μ to each point of the Teichmuller space $\zeta(\mu)$. Over a hyperbolic structure of $g \geq 2$ the endowed diffeomorphism $\mathcal{D}:\mu \rightarrow \mathcal{G}$ the existence of uniformization theorem can be found at^[22],

$$\begin{cases} 1 \text{ point in Teichmuller space} \\ 2 \text{ points for unit disk and complex plane} \end{cases} \text{ for contractible diffeomorphism in each case}$$

Take any linear injective map with the functor of a Kernel whilst the vector ρ taking through a projective space $\mathbb{P}(\rho)$ then the embedded mapping for that associated functor represents a channel over automorphisms where any such map defines a transition to a corresponding vector space σ via,

$$\mathbb{P}(map): \mathbb{P}(\rho) \rightarrow \mathbb{P}(\sigma) \text{ such that } map: \rho \rightarrow \sigma \text{ satisfies at first}$$

Thus for \mathbb{P}^n for \mathbb{P}^4 the non-trivial topology of compact Ricci flat Kähler takes reign while the high degrees analogy for \mathbb{P}^7 is generalized in canonical and terminal singularities or the non-singular approaches for the quintic 3-fold structures for the associated Hodge lemma $\partial\bar{\partial}$ with the Kähler potential $\partial\bar{\partial}_\rho$ having the imaginary coefficients $2^{-1}i$ for $(p,q) \equiv (1,1)^+$ form has the Kodaira embedding dimensions with the Hodge diamonds $h^{1,1}$, $h^{2,1}$ represents the multi-homogeneous polynomial \mathcal{E}_p structured for CY space through^[2],

$$X_c(\mathcal{C}) = \begin{bmatrix} Q_1^1 & Q_2^2 & \dots & \dots & Q_1^k \\ Q_2^1 & Q_2^2 & \dots & \dots & Q_2^k \\ \vdots & \vdots & \ddots & \dots & \vdots \\ \vdots & \vdots & \dots & \ddots & \vdots \\ Q_l^1 & Q_l^2 & \dots & \dots & Q_l^k \end{bmatrix}$$

Where for p^{th} polynomial degree of Q_n^p goes from 1 to k over the n^{th} factor of \mathcal{E}_n from 1 to l with the corresponding moduli being^[2],

$$\begin{array}{ll} \text{Kähler moduli} & h^{1,1} = 1 \\ \text{Complex moduli} & h^{2,1} = 101 \quad \mathcal{X} = 2(h^{1,1}, h^{2,1}) \\ \text{CY 3-fold} & h^{1,1}, h^{2,1} \end{array}$$

All for Kähler, complex, CY spectrum emphasizing 3-folds being the non-trivial CY appearing in supersymmetric string theories with \mathcal{X} being the Euler's characteristics. The Kähler not being the hyperbolic hypercomplex – we should follow the positive geometry having the associated topology having the positive sectional curvature. That's being reduced in dimensions or degrees when we get a Riemann metric for any 3-manifold taking the same positive sectional curvatures – The elliptization conjecture can be proved as^[1,23],

for that related 3 – manifold its homeomorphic to a 3 – sphere

Being analogous to the uniformization for 2-dimensions where the associated geometry can be any one – Elliptic, Hyperbolic or Euclidean, the 3-dimensions are pervasive where it's difficult to assign a single curvature for which decomposition to that 3-manifold each having one of the 8-geometries in essence linked to the elliptization and geometrization covering the 8-geometries of Thurston as expressed over Point Stabilizers (PS)^[24],

- [1] Type – Euclidean
- [2] Type – Spherical
- [3] Type – Hyperbolic
- [4] Type – Universal cover
- [5] Type – $\mathbb{S}^2 \times \mathbb{R}$
- [6] Type – $\mathbb{H}^2 \times \mathbb{R}$
- [7] Type – Solv
- [8] Type – Nil

For the simply connected Riemannian 3-manifold with the metric g_R through the temporal evolution, a curve shortening flow occurs with the associated *Ricci* curvature such that the time T begins from 0 to I for I less than infinity with the formulations^[23],

for every $I < \infty$ there exists $T \in 0, I$

$$\left\{ \begin{array}{l} \text{The } -1 \text{ curvature expands} \\ \text{The } +1 \text{ curvature contracts} \\ \text{Then the Riemann metric converges to } +1 \text{ curvatures i.e. Ricci flow} \end{array} \right.$$

Definition: This Ricci flows over the Riemann metric leads to shrinking cycles where the space splits into two by surgery after developing the singularity.

Discussion:

Covering the most essential aspects of geometry and curvatures taking place with groups and other associated degrees from general relativity to topological string theory with the inclusion of a bit perspectives of string field theory – analyzation and classification have been computed over classifiers segregating each geometry with topologies in the scope of their importance and requirements in the fields of theoretical physics and mathematical physics. The general theory of relativity and the statistics of particles with the coherent norm of strings as quantum gravity over renormalizations to cope with ultraviolet divergences are mentioned here along with the most basic axioms of special relativity. The non-trivial topology of strings that is CY manifold with an added perspective of CY 3-folds discussions is being made from hypercomplex to hyperbolic complex structures as and where required. Several proofs are associated with the conjectures being computed with simple mathematics for the understanding in a relatively easier manner.

References:

- [1] Bhattacharjee, D. (2022i). Establishing equivalence among hypercomplex structures via Kodaira embedding theorem for non-singular quintic 3-fold having positively closed $(1,1)$ -form Kähler potential $i\partial\bar{\partial}^*p$. *Research Square*. <https://doi.org/10.21203/rs.3.rs-1635957/v1>
- [2] Bhattacharjee, D. (2022j). Generalization of Grothendieck duality over Serre duality in deg_n Cohen-Macaulay schemes representing Calabi–Yau 3-fold on Bogomolov–Tian–Todorov Theorem. *Research Gate*. https://www.researchgate.net/publication/361452066_Generalization_of_Grothendieck_duality_over_Serre_duality_in_Cohen-Macaulay_schemes_representing_Calabi-Yau_3-fold_on_Bogomolov-Tian-Todorov_Theorem
- [3] Bhattacharjee, D., Amani, D., Behera, A. K., Sadhu, R., & Das, S. (2022c). Young Sheldon’s Rough Book on Strings – Decomplexifying Stuff. *Authorea Preprint*. <https://doi.org/10.22541/au.165057530.08903120/v1>
- [4] Bhattacharjee, D. (2022k). Homotopy Group of Spheres, Hopf Fibrations & Villarceau Circles. *EasyChair Preprint No. 7959*. <https://easychair.org/publications/preprint/WQH8>
- [5] Bhattacharjee, D. (2022r). Rigorously Computed Enumerative Norms as Prescribed through Quantum Cohomological Connectivity over Gromov – Witten Invariants. *TechRxiv*. <https://doi.org/10.36227/techrxiv.19524214.v1>
- [6] Bhattacharjee, D. (2022c). Atiyah – Hirzebruch Spectral Sequence on Reconciled Twisted K – Theory over S – Duality on Type – II Superstrings. *Authorea Preprint*. <https://doi.org/10.22541/au.165212310.01626852/v1>
- [7] Bhattacharjee, D. (2022a). A Coherent Approach Towards Quantum Gravity. *TechRxiv*. <https://doi.org/10.36227/techrxiv.19785880.v1>
- [8] Rovelli, C. (2007). *Quantum Gravity (Cambridge Monographs on Mathematical Physics)* (1st ed.). Cambridge University Press.
- [9] Harikant, A., Singha Roy, S., & Bhattacharjee, D. (2021). Computing the temporal intervals by making a Throne-Morris wormhole from a Kerr black hole in the context of $f(R,T)$ gravity. *International Journal of Scientific Research and Management*, 9(07), 72–92. <https://doi.org/10.18535/ijstrm/v9i07.aa01>

- [10] Bhattacharjee, D. (2020c). Solutions of Kerr Black Holes subject to Naked Singularity and Wormholes. *Authorea Preprint*. <https://doi.org/10.22541/au.160693414.46356832/v1>
- [11] Niven, R. K. (2005). Exact Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac Statistics. *arXiv:Cond-Mat/0412460v2 [Cond-Mat.Stat-Mech]*. <https://doi.org/10.1016/j.physleta.2005.05.063>
- [12] Bhattacharjee, D. (2022x). Uniqueness in Poincaré-Birkhoff-Witt Theorem over Algebraic Equivalence. *Authorea Preprint*. <https://doi.org/10.22541/au.165511635.53854231/v1>
- [13] F., L., & S. (2013). *The Feynman Lectures on Physics*. <https://www.feynmanlectures.caltech.edu/>. Retrieved June 23, 2022, from <https://www.feynmanlectures.caltech.edu/>
- [14] Bhattacharjee, D. (2020a). Formulation of a Catenoid Structured Wormhole without any Exotic Matter. *International Journal for Research in Applied Science and Engineering Technology*, 8(7), 1248–1257. <https://doi.org/10.22214/ijraset.2020.30480>
- [15] Fleisch, D. (2008). *A Student's Guide to Maxwell's Equations (Student's Guides) 1st Edition* (1st ed.). Cambridge University Press.
- [16] G. (2022). *Elegant Universe (05) by Greene, Brian [Paperback (2005)]*. Vintage s, Paperback(2005).
- [17] *[The Road to Reality] [By: Penrose, Roger] [February, 2006]*. (2004). Vintage Books.
- [18] Munkres, J. R. (2022). *Topology* (2nd ed.). Pearson.
- [19] Kaku, M. (1995). *Hyperspace: A Scientific Odyssey Through Parallel Universes, Time Warps, and the 10th Dimension* (Edition Unstated ed.). Anchor.
- [20] Labbi, M. L. (2007). On Gauss-Bonnet Curvatures. *Symmetry, Integrability and Geometry: Methods and Applications*. <https://doi.org/10.3842/sigma.2007.118>
- [21] PINTO-NETO, N., MARTIN, J., & SOARES, V. D. (2005). GREEN FUNCTIONS FOR TOPOLOGY CHANGE. *International Journal of Modern Physics A*, 20(11), 2393–2397. <https://doi.org/10.1142/s0217751x05024675>
- [22] Leininger, C., & Schleimer, S. (2014). Hyperbolic spaces in Teichmüller spaces. *Journal of the European Mathematical Society*, 16(12), 2669–2692. <https://doi.org/10.4171/jems/495>
- [23] Bhattacharjee, D. (2021c). The Gateway to Parallel Universe & Connected Physics. *Preprints*. <https://doi.org/10.20944/preprints202104.0350.v1>
- [24] Grützmacher, S. (2015). The 8 geometries of 3-manifolds. *University of Heidelberg - Riemannian Geometry Seminar*. <https://www.mathi.uni-heidelberg.de/~lee/Sven06.pdf>