

Predicting the three-dimensional age-depth field of an ice rise

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Key Points:

- First three-dimensional simulations of the stratigraphy of an ice rise allowing comparison of model results with radar observations.
- Choice of the Glen's flow law exponent influences deformation in the grounding zones.
- Reduction in surface elevation at the divide relative to observations points at missing processes in the model such as anisotropy.

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20 Abstract

21 Ice rises situated around the perimeter of Antarctica buttress ice flow and contain
 22 information about the past climate and changes in flow regime. Moreover, ice rises con-
 23 tain convergent and divergent flow regimes, and both floating and grounded ice over com-
 24 paratively small spatial scales, meaning they are ideal locations to study ice-flow dynam-
 25 ics. Here, we introduce a new modelling framework that permits the comparison between
 26 modelled and observed stratigraphy. A thermo-mechanically coupled, isotropic, Stokes
 27 ice flow model with a dynamic grounding line is used (Elmer/Ice). The result is the sim-
 28 ulated age-depth field of a three-dimensional, steady-state ice rise which is dynamically
 29 coupled to the surrounding ice shelf. Applying the model to Derwael Ice Rise, results
 30 show a good match between observed and modelled stratigraphy over most of the ice rise
 31 and predict approximately 8000 year old ice at a depth of 95 %. Differences in the pre-
 32 diction of age between simulations using Glen’s flow law exponents of $n = 3$ and $n =$
 33 4 are generally small (< 5 % over most areas). In the ice rise shear zones, large differ-
 34 ences in shear strain rates in the velocity direction are found between the $n = 3$ and
 35 the $n = 4$ simulations. Our simulations indicate that a Glen’s flow law exponent of $n =$
 36 4 may be better suited when modelling ice rises due to a steady-state geometry which
 37 is closer to the observed geometry. Our three-dimensional modelling framework can eas-
 38 ily be transferred to other ice rises and has relevance for researchers interested in ice core
 39 dating and understanding ice-flow re-organisation.

40 Plain Language Summary

41 Ice rises are features which form in coastal Antarctica when the ice shelf comes into
 42 contact with the bathymetry. These features provide a backstress on the ice shelf and
 43 can influence grounding line position. We simulate an ice rise in East Antarctica called
 44 Derwael Ice Rise, outlining the steps necessary to model the three-dimensional stratig-
 45 raphy of an ice rise and compare the modelled stratigraphy with observed stratigraphy
 46 derived from radar measurements. Comparisons between the observed and modelled stratig-
 47 raphy allow us to validate boundary conditions and the parameterisations used in our
 48 model. This work is relevant as a blueprint for simulating other ice rises for those inter-
 49 ested in comparison with ice core records, and investigating ice rises formation and evo-
 50 lution.

51 1 Introduction

52 Ice rises form where ice shelves ground locally on topographic highs in the bathym-
 53 etry and are important in coastal Antarctic ice flow dynamics as they regulate the flow
 54 of ice towards the ocean (Favier & Pattyn, 2015; Favier et al., 2016; Henry et al., 2022).
 55 Moreover, ice rises are valuable as a climate archive because they often provide high-resolution
 56 and undisturbed records throughout the Holocene. Ice-core drill sites are often located
 57 at local summits to avoid lateral flow. However, it is a significant challenge to predict
 58 the age-depth fields prior to drilling. This is due to strong variations in surface mass bal-
 59 ance (SMB, Cavitte et al. (2022)) and also because the ice-flow regimes change over a
 60 few tens of kilometres. Divide flow at the summit (where arches in the internal stratig-
 61 raphy may form) turns into flank flow, and finally to the grounding zone where coupling
 62 with the surrounding ice shelves takes place. Compared to Antarctica’s interior, ice rises
 63 at the coast are comparatively easy to reach and consequently a number of them, Der-
 64 wael Ice Rise being one of them, have been densely surveyed with radar to image the isochronal
 65 stratigraphy. This enables the comparison of model predictions across various flow regimes
 66 with observations which can help calibrate model parameters such as Glen’s flow law ex-
 67 ponent, the fundamental constitutive relation for ice flow.

68 The non-Newtonian flow of ice (J. W. Glen, 1955; Weertman, 1983; Budd & Jacka,
 69 1989) results in Raymond arches (Raymond, 1983) which form in the stratigraphy un-
 70 der the ice-rise divides and have been used to estimate how stationary ice-divide flow is.
 71 This effect can be dampened, for example, by along-ridge flow or changing conditions,
 72 thus inhibiting their formation. Under a changing climate, the geometry of an ice rise
 73 often changes, thereby causing a change in the isochronal structure (Nereson et al., 1998;
 74 Martín et al., 2009). The onset of stability of an ice rise is indicated by the amplitude
 75 of the Raymond arches and a change in the size of an ice rise is indicated by the migra-
 76 tion of Raymond arches visible in the stratigraphy as side arches or tilted anticline stacks.
 77 Simulations of the stratigraphy of specific ice rises have thus far been performed in two
 78 dimensions (Martín et al., 2006, 2009, 2014; Drews et al., 2015; Goel et al., 2018), with
 79 Gillet-Chaulet and Hindmarsh (2011) performing simulations of the stratigraphy of an
 80 idealised ice rise in three dimensions without the inclusion of the surrounding ice shelf.

81 In this paper, we build on previous ice-rise modelling studies (Martín et al., 2009;
 82 Drews et al., 2015; Schannwell et al., 2019, 2020) and extend them by introducing a mod-
 83 elling framework that allows us to model ice rises including the surrounding ice shelves
 84 and their stratigraphy in three dimensions. This not only permits the prediction of the
 85 stratigraphy, but also accounts for three-dimensional effects that are of importance for
 86 comparisons with radar observations and ice cores. Whilst having proven important in
 87 the development of an understanding of Raymond arches, the two-dimensional studies
 88 do not allow for along-ridge flow. Studies investigating the observed stratigraphy in shear
 89 zones (Franke et al., 2022) and zones of convergence (Bons et al., 2016) have been per-
 90 formed, but a comparison between observed and modelled stratigraphy in such settings
 91 has not yet been performed. In idealised simulations (Hindmarsh et al., 2011; Gillet-Chaulet
 92 et al., 2011), it has been shown that along-ridge flow has a dampening effect on Raymond
 93 arch evolution. Where these simulations lack, however, is in the use of idealised bound-
 94 ary forcing conditions, which do not sufficiently produce the differing flow regime con-
 95 ditions on the stoss and lee sides of an ice rise.

96 The introduction of our new modelling framework provides a blueprint for mod-
 97 elling a real-world ice rise in three dimensions using the thermo-mechanically coupled
 98 model Elmer/Ice (Gagliardini et al., 2013) in order to predict the age field ice-rise wide.
 99 We investigate how robust those results are compared to observations with two Glen’s
 100 flow law exponents. We choose to compare simulations using the typical exponent of $n =$
 101 3 with simulations using an exponent of $n = 4$, closer to the value of $n = 4.1 \pm 0.4$
 102 found to work best for Antarctic ice shelves (Millstein et al., 2022) and similar values
 103 suggested by Bons et al. (2018) for Greenland. The conversion from using a Glen’s flow
 104 law with an exponent of $n = 3$ to an exponent of $n = 4$ is made using an initial scalar
 105 stress estimate along with simulations for the evaluation of an appropriate Arrhenius pre-
 106 factor for $n = 4$.

107 The three-dimensional, steady-state simulations presented here have relevance for
 108 comparisons with ice cores and in the context of understanding the link between isochronal
 109 structures and changes in ice geometry and external forcing. Steady-state simulations
 110 allow the deduction of changes due to misfits and provide an important step towards the
 111 use of ice rises as a constraint for paleo ice-sheet simulations. This study not only suc-
 112 cessfully demonstrates three-dimensional modelling to bridge Stokes models with observed
 113 radar stratigraphy, but also delves into the implications of model parameter choice by
 114 exploiting variables in Glen’s flow law.

115 2 Derwael Ice Rise

116 Derwael Ice Rise has a grounded area of roughly 1050 km^2 and is an isle-type ice
 117 rise with a ridge divide. The grounded area has a maximum width of roughly 35 km per-
 118 pendicular to the predominant flow direction of the ice shelf. The ice rise has a maxi-

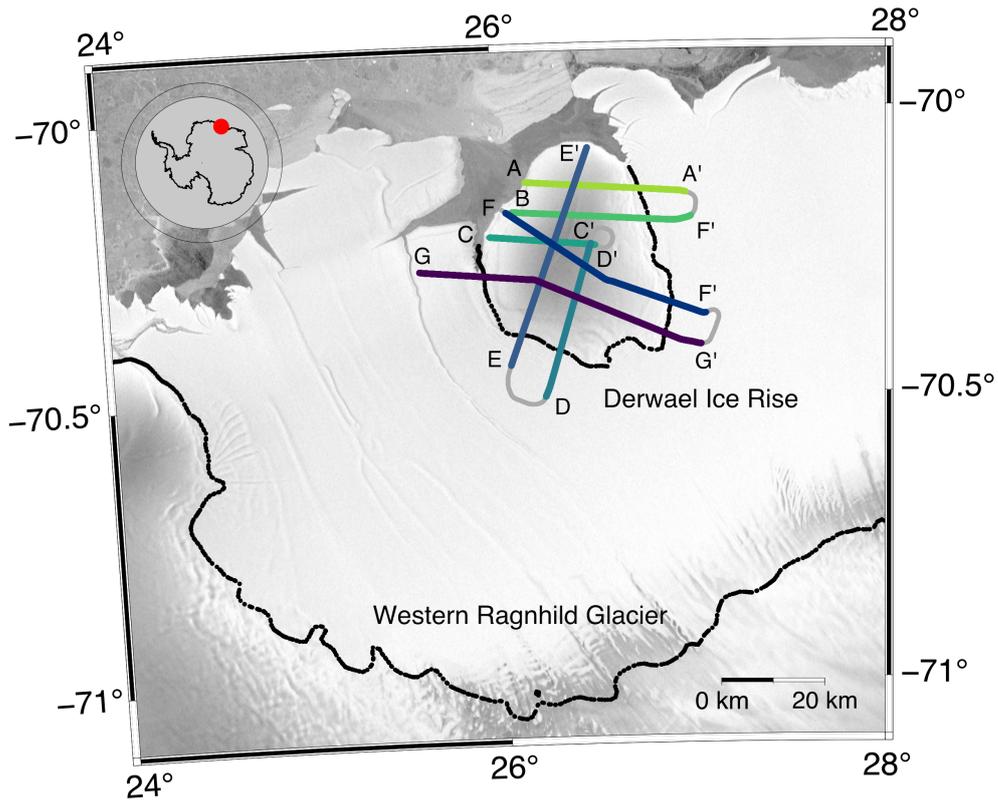


Figure 1. The location of Derwael Ice Rise within the Roi Baudouin Ice Shelf in East Antarctica. The coloured line segments $A - A'$ to $G - G'$ indicate the locations of radar measurements taken using airborne radar. The continental and ice rise grounding lines are indicated by the black lines. The RADARSAT mosaic (Jezek, 2003) is shown in the background, and the grounding line (black dots) is from Morlighem et al. (2020).

119 mum ice thickness of roughly 630 m with an estimated accuracy of 5 % (Morlighem et
 120 al., 2020) and is thickest in the south of the ice rise, where there is convergence of flow
 121 from the ice rise and the ice shelf. The maximum ice thickness at the ridge divide is roughly
 122 540. We choose Derwael Ice Rise because of the availability of radar data across the ice
 123 rise divide and the shear margins, and also because Derwael Ice Rise is close to steady-
 124 state, perhaps with some current thinning (Drews et al., 2015; Callens et al., 2016). Der-
 125 wael Ice Rise has well expressed isochrone arches beneath the ridge divide. A peculiar-
 126 ity is, that arches (referred to as side arches later on) also occur in the south-eastern flanks
 127 close to the divide (Drews et al., 2015). An ice rumple is located in the north-western
 128 corner of the domain.

129 3 Methods

130 The model setup is based on Henry et al. (2022), and here we extend the frame-
 131 work to real-world geometries. In the following sections, we describe the required mod-
 132 ifications to accomplish this. We use the finite element software Elmer/Ice (Gagliardini
 133 et al., 2013) to solve the Stokes equations. Here, we describe the coupled equations, model
 134 parameters boundary conditions and mesh resolution.

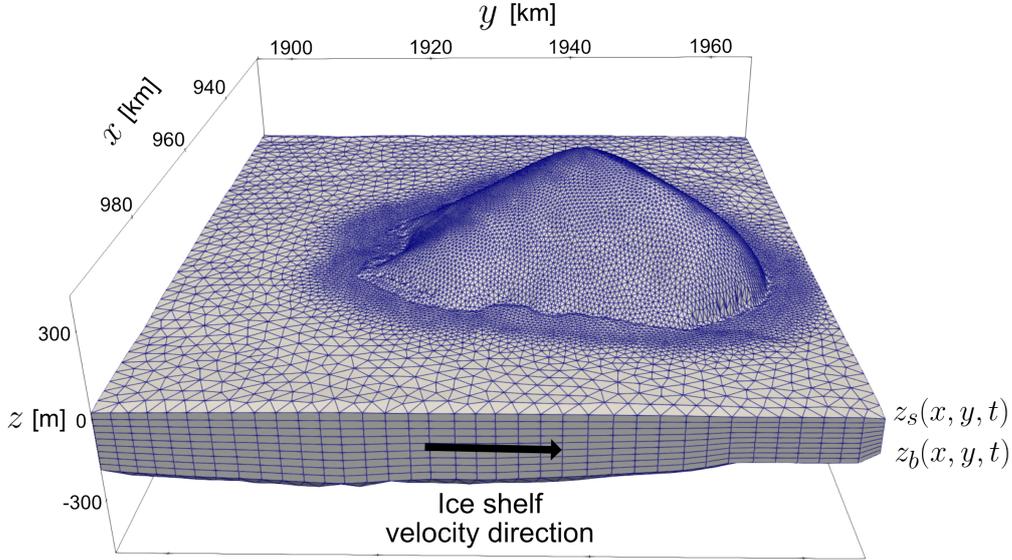


Figure 2. The model set up with horizontal distances in Antarctic polar stereographic projection. The area encompassing the ice rise has a characteristic resolution of 500 m and the surrounding area has a resolution of 2000 m. The upper ice surface is denoted by $z_s = z_s(x, y, t)$ and the lower ice surface by $z_b = z_b(x, y, t)$, where x and y are the horizontal directions and z is the vertical direction relative to sea level. Note: for visualisation, the vertical is scaled by a factor of 30.

Table 1. List of parameters used in the simulations

Parameter	Symbol	Value	Unit
Basal friction exponent	m	1/3	
Local ocean density	ρ_w	1000	kg m ⁻³
Ice density	ρ_i	900	kg m ⁻³
Gravity	g	9.8	m s ⁻²
Universal gas constant	R	8.314	mol ⁻¹ K ⁻¹
Geothermal heat flux	ϕ_q	50	mWm ⁻²
Basal melt parameter	b_0	0.95	ma ⁻¹

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3.1 Governing equations

The Stokes equations,

$$\nabla \cdot (\boldsymbol{\tau} - P\mathbf{I}) + \rho_i \mathbf{g} = 0, \quad (1)$$

describe the flow of ice, where $\boldsymbol{\tau}$ is the deviatoric stress tensor, P is the pressure, \mathbf{I} is the identity matrix, ρ_i is the ice density and $\mathbf{g} = -g\hat{\mathbf{e}}_z$ is the gravitational acceleration. The ice is subject to an incompressibility condition,

$$\nabla \cdot \mathbf{u} = 0. \quad (2)$$

The Glen's power flow law,

$$\boldsymbol{\tau} = 2\eta\dot{\boldsymbol{\epsilon}}, \quad (3)$$

describes the nonlinear dependence between the strain rate tensor, $\dot{\boldsymbol{\epsilon}}$, and the deviatoric stress tensor. The effective viscosity, η , is

$$\eta = \frac{1}{2}A(T)^{-1/n}\dot{\epsilon}_e^{(1-n)/n}, \quad (4)$$

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where $A(T)$ is the ice fluidity which is dependent on temperature, T , and is described in detail below. The effective strain rate, $\dot{\epsilon}_e^2$, is the square of the second invariant of the strain rate tensor, $\dot{\boldsymbol{\epsilon}}$. As in Gagliardini et al. (2013), the temperature of the ice evolves subject to

$$\rho_i c_v \left(\frac{\delta T}{\delta t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot (\kappa \nabla T) + \mathbf{D} : \boldsymbol{\sigma}, \quad (5)$$

where $:$ is the double inner product,

$$c_v = 146.3 + 7.253T \quad (6)$$

is the specific heat capacity of the ice and

$$\kappa = 9.828 \exp(-5.7 \times 10^{-03}T) \quad (7)$$

is the thermal conductivity (Ritz, 1987). The temperature is coupled to the Glen's flow law using an Arrhenius law

$$A(T, p) = EA_0 \exp(-Q/RT), \quad (8)$$

where A_0 is a constant pre-factor, Q is the activation energy and R is the universal gas constant. The Arrhenius law is multiplied by a constant, E , called an enhancement factor, in order to obtain an optimal coefficient in the Arrhenius law. The combination of the parameters A_0 and E are used in ice sheet modelling to account for effects such as grain size, crystal orientation, impurities, porosity and water content. An exploration of the influence of each process is beyond the scope of this study, but we will note here that processes which soften ice cause either an increase in the parameter A_0 or E . For calculating the equivalent Arrhenius factor for a Glen's flow law exponent of $n = 3$, we take a similar approach to Zeitz et al. (2020) and use a first estimate of the stress magnitude of $[\tau_0] = 0.25 \times 10^6$ Pa, so that

$$A_0|_{n=4} \exp(-Q|_{n=4}) = \frac{A_0|_{n=3} \exp(-Q|_{n=3})}{[\tau_0]}. \quad (9)$$

The first estimate of τ_0 is to compensate for the multiplication of an additional deviatoric stress tensor in Glen's flow law. The upper surface temperature is set equal to the temperature field data (Comiso, 2000). Initially, a linear temperature profile from the lower ice surface to the upper ice surface is prescribed. During transient simulation, the upper and lower surface temperatures evolve subject to a Neumann boundary condition.

During initialisation, the lower ice surface temperature is prescribed to be the pressure melting point temperature,

$$T_p = 273.15 - \beta \rho_i g (z_s - z). \quad (10)$$

Here, β denotes the Clausius-Clapeyron constant, $\beta = 9.8 \times 10^{-8} \text{ K Pa}^{-1}$ (Zwinger et al., 2007). In order to solve for the isochronal stratigraphy of the ice, the age of the ice is solved according to

$$\frac{\partial \psi}{\partial t} + \mathbf{u} \cdot \nabla(\psi) = 1 \quad (11)$$

140 where ψ is the age of the ice (Zwinger & Moore, 2009). Eq. 11 is solved using a semi-
141 Lagrangian scheme implemented in Elmer/Ice (Martín & Gudmundsson, 2012).

The upper ice surface, $z = z_s(x, y, t)$, and the lower ice surface, $z = z_b(x, y, t)$, evolve subject to

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) (z - z_s) = \dot{a}_s, \quad (12)$$

and

$$\left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) (z - z_b) = \dot{a}_b, \quad (13)$$

respectively, where $\dot{a}_s = \dot{a}_s(x, y)$ is the ice-equivalent SMB. The basal melt rate, $\dot{a}_b = \dot{a}_b(x, y)$, is set to a suitable constant of 0.95 m a^{-1} which resulted in minimal adjustment of ice shelf thickness and grounding line position and is close to the average spatial value of 0.8 m a^{-1} across the Roi Baudouin Ice Shelf (Drews et al., 2020). The SMB, \dot{a}_s , is described in further detail below. Where ice is in contact with the bed, a non-linear Weertman friction law (Weertman, 1957) is used,

$$\boldsymbol{\tau}_b = -C |\mathbf{u}_b|^{m-1} \mathbf{u}_b, \quad (14)$$

142 where $\boldsymbol{\tau}_b$ is the basal shear stress, C is a constant friction coefficient, \mathbf{u}_b is the velocity
143 tangential to the bed, and m is the friction law exponent and has the value $m = 1/3$
144 in all simulations.

145 For all our simulations, we use a horizontal resolution of 500 m in the area encom-
146 passing the ice rise up to a distance from the grounding line of 5000 m and the surround-
147 ing area has a resolution of 2000 m (Fig. 2). In the vertical, the mesh is made up of 10
148 layers. The higher resolution is needed in order to better resolve the stratigraphy of the
149 ice rise. The Elmer/Ice grounding line implementation *Discontinuous* is used.

150 **3.2 Observational data, initial conditions and boundary conditions**

151 **3.2.1 Observational stratigraphy**

152 Airborne radar data were acquired in the 2018/19 Antarctic field season as part
153 of the CHIPSM survey using the Polar 6 aircraft of the Alfred Wegener Institute with
154 an ultrawideband radar (MCoRDS v5) and eight-element fuselage antenna array oper-
155 ating in the 150–520 MHz frequency range. Details on data processing and tracing of
156 isochronal internal layers is laid out in (Koch, Drews, Franke, et al., n.d.), which use the
157 same data set. Dating of the two shallowest internal layers along the radar profiles is based
158 on an ice-core depth scale (Philippe et al., 2016) under the assumption of a steady-state
159 age-depth relation (Koch, Drews, Franke, et al., n.d.). This yields a total of seven pro-
160 files across the ice rise (Fig. 1).

161 In order to make comparisons between the modelled isochronal stratigraphy and
162 the observed internal reflection horizons possible, a density adjustment needs to be made.
163 This can be done either by adjusting the modelled isochrone elevation to match the den-
164 sity profile of the real-world ice rise or vice versa. We choose the latter. The adjustment

Table 2. List of simulations

Simulation	n	E	Pre-factor 1 [Pa ⁻ⁿ a ⁻¹]	Pre-factor 2 [Pa ⁻ⁿ a ⁻¹]
n3E0.2	3	0.2	1.258×10^7	6.046×10^{22}
n3E0.4	3	0.4	1.258×10^7	6.046×10^{22}
n3E0.5	3	0.5	1.258×10^7	6.046×10^{22}
n3E0.6	3	0.6	1.258×10^7	6.046×10^{22}
n3E0.8	3	0.8	1.258×10^7	6.046×10^{22}
n3E1.0	3	1.0	1.258×10^7	6.046×10^{22}
n4E1.2	4	1.2	5.032×10^7	2.419×10^{23}
n4E1.6	4	1.6	5.032×10^7	2.419×10^{23}
n4E1.8	4	1.8	5.032×10^7	2.419×10^{23}
n4E2.0	4	2.0	5.032×10^7	2.419×10^{23}
n4E2.4	4	2.4	5.032×10^7	2.419×10^{23}
n4E2.8	4	2.8	5.032×10^7	2.419×10^{23}

165 results in isochrone elevations equivalent to a constant density of 900 kg m^{-3} and is cal-
 166 culated according to the density profile of Derwael Ice Rise in Callens et al. (2014). Through-
 167 out the paper, when referring to depth below surface in relation to observations, these
 168 are relative to the BedMachine surface elevation, adjusted to an equivalent of 900 kg m^{-3}
 169 as opposed to the 917 kg m^{-3} assumed in BedMachine. On the other hand, when refer-
 170 ring to depth below surface in relation to model results, depths are relative to the steady-
 171 state modelled surface using a density of 900 kg m^{-3} . Note that this choice does not have
 172 an effect on the final comparison between the modelled and observed isochrones.

173 **3.2.2 Surface and basal mass balances, velocity, bed and ice geometry**

174 Over the ice rise, we derive the SMB along transects from the shallow radar stratig-
 175 raphy using the standard approach of the shallow layer approximation (Waddington et
 176 al., 2007). Isochrones are dated using an ice core drilled at the summit (Philippe et al.,
 177 2016). Details of this are presented in (Koch, Drews, Franke, et al., n.d.) accompanied
 178 by the dataset (Koch, Drews, Muhle, et al., n.d.). We interpolated between radar tran-
 179 sects using an interpolation scheme and merge the SMB field at the ice-rise edges with
 180 reanalysis data from RACMO2.3p1 mean annual SMB from the years 1979-2014 (van
 181 den Broeke, 2019). Including the SMB estimates from radar observations is a critical step
 182 in the analysis because the reanalysis data are too coarsely resolved on the ice rise. In
 183 order to avoid model drift due to uncorrected offsets in the SMB field, we correct the
 184 SMB field by subtracting it by the rate of change of the surface elevation after 50 years
 185 of simulation time.

186 At the domain boundary on the oceanward side of the ice rise, the ice is allowed
 187 to flow subject to hydrostatic pressure. At all other boundaries, depth-independent fluxes
 188 are prescribed and are derived from observed velocities (Rignot & Scheuchl., 2017). The
 189 bed elevation and the initial ice geometry is prescribed using BedMachine Antarctica data
 190 (Morlighem et al., 2020). When comparing BedMachine bed elevation with observations
 191 from our radar survey, we found significant mismatches of roughly 150 m in the north-
 192 eastern corner of Derwael Ice Rise (Fig. 4). This is surprising given that the radar sur-
 193 vey is part of the BedMachine dataset. We see some interpolation artefacts where the
 194 grounded ice bed elevation dataset is merged with the bed elevation data below float-
 195 ing ice.

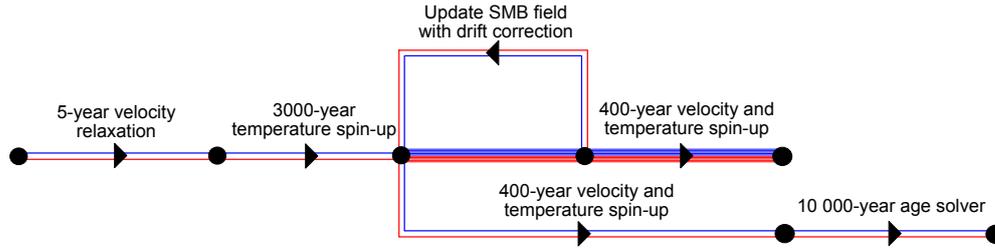


Figure 3. The schematic shows the spin-up procedure for the simulations in Table 2. The blue lines refer to the $n = 3$ simulations and the red lines refer to the $n = 4$ simulations. The black dots indicate the points when the simulation was restarted with a change in model set up as indicated by the schematic labels. Further details regarding the sequence of steps are given in Section 3.3.

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3.3 Model spin-up procedure

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In order to model the three-dimensional isochronal stratigraphy of an ice rise, the following steps are taken to spin-up the model. The details are as follows and can be seen in Fig. 3.

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Step 1: Simulate the ice rise for 5 years with the Stokes, temperature, and upper and lower free surface solvers on for Glen’s flow law exponents of $n = 3$ and $n = 4$.

Step 2: Spin up the temperature for 3000 years with the Stokes and free surface solvers off.

Step 3: Simulate with the chosen set of parameters (Table 2) for 400 years.

Step 4: Choose the optimal $n = 3$ and $n = 4$ simulation based on the least volume change (Fig. 6).

Step 5: Compute SMB from model drift after allowing the surface elevation to evolve for 50 years. The SMB is adjusted using the model drift over the grounded area and is incorporated with the combined stratigraphy-derived and RACMO2.3p1 SMB field. A Gaussian filter is applied to remove steep gradients.

Step 6: Simulate with the temperature, free surface and Stokes solvers activated for 400 years.

Step 7: Run the age solver with the Stokes, free surface and temperature solvers off for 10000 years.

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4 Results

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4.1 Model parameter choice and applied surface mass balance

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In contrast to the SMB field from the RACMO2.3p1 simulation, the highest values of the stratigraphy-derived and drift-corrected SMB are concentrated in the centre of the ice rise (Fig. 5 in the Supporting Information). Here, the differences in spatial variation before and after the drift correction are evident, with the drift-corrected SMB being concentrated more towards the west of the ice rise. RACMO2.3p1 data shows higher values on the eastern and northern sides of the ice rise, whereas the stratigraphy-derived and drift-corrected SMB has higher values in the centre and on the south-western side of the ice rise. The results are close to mean SMB values of 0.47 ± 0.02 m w.e. a^{-1} (for comparison with our results, this is equivalent to 0.52 ± 0.02 m a^{-1} assuming an ice density of 900 kg m^{-3}) for the period 1816–2011 found by (Philippe et al., 2016), derived from an ice core at the summit of Derwael Ice Rise.

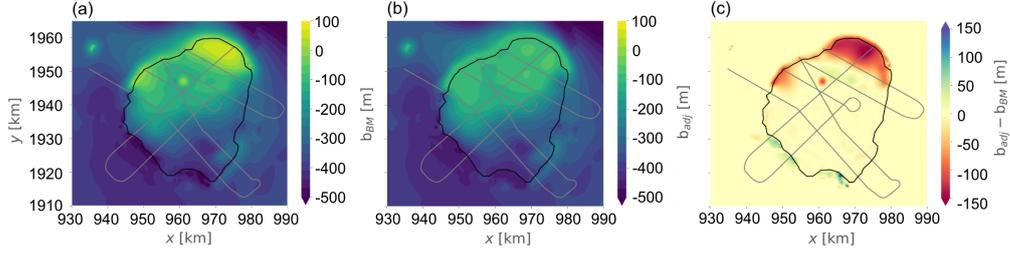


Figure 4. The BedMachine Antarctica bed elevation is shown in (a) and the adjusted bed elevation after re-interpolation and smoothing of unphysical anomalies is shown in (b). The difference between the adjusted bed elevation and the original BedMachine bed elevation is shown in (c). The grey lines show the location of radargrams with which the simulations are compared. The bed elevation data from these radar lines has been used in the BedMachine data.

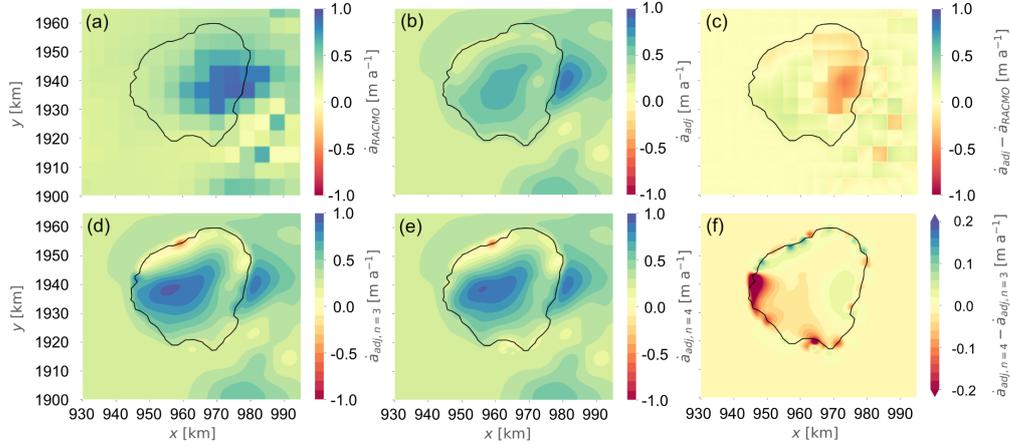


Figure 5. Surface mass balance (SMB) (a) based on RACMO2.3p1 data; (b) the adjusted product from a combination of stratigraphy-derived data in the grounded area and RACMO2.3p1 data in the surrounding area; (c) difference between the adjusted and the RACMO2.3p1 SMB. The SMB in (d) and (e) use a drift correction made with the $\partial z_s / \partial t$ field after a simulation time of 50 years for the $n = 3$ and the $n = 4$ simulations, respectively and (f) shows the difference in SMB between the $n = 3$ and $n = 4$ simulations for the stratigraphy-derived and drift-corrected SMB.

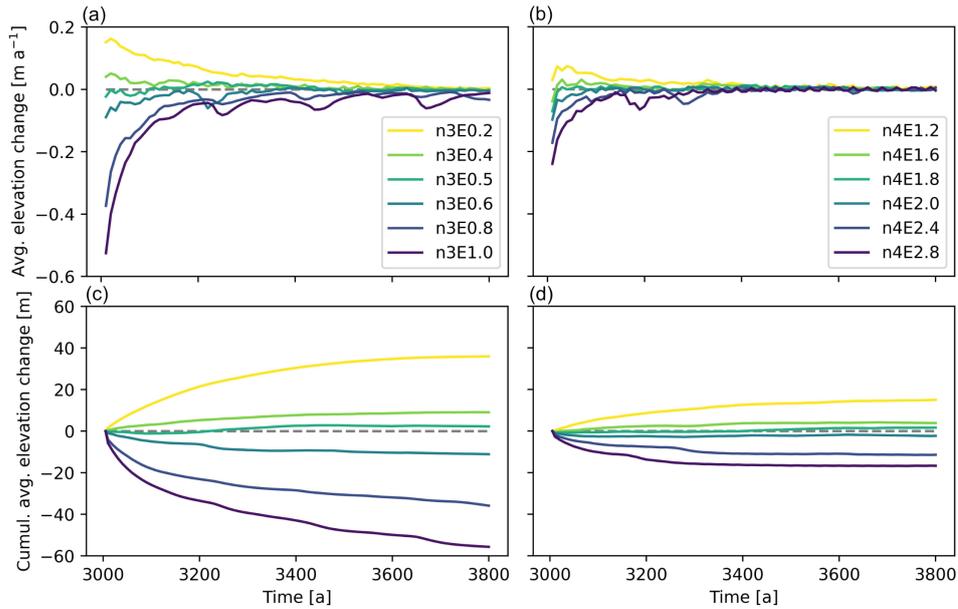


Figure 6. Elevation changes for different simulations: (a) and (b) show the average elevation changes, and (c) and (d) show the cumulative elevation changes for Glen’s flow law exponents of $n = 3$ and $n = 4$, respectively, for varying enhancement factors. Legends indicate the different model runs in colour.

228 In order to find the optimal combination of parameters in the simulations of Der-
 229 wael Ice Rise, an ensemble of simulations for various enhancement factors were performed.
 230 Here we note that in this work we take the enhancement factor to simply be a constant
 231 multiplier that adjusts the Arrhenius factor, A_0 and is equivalent to adjusting the Ar-
 232 rhenius pre-factor itself. As a performance evaluation metric in order to find the opti-
 233 mal enhancement factor, we use the change in surface elevation with respect to time across
 234 the grounded ice (Fig. 6), assuming that Derwael Ice Rise is in steady state. We found
 235 that an enhancement factor of $E = 0.5$ (simulation $n3E0.5$) produced an ice rise with
 236 the least cumulative volume change in the $n = 3$ simulations. For the $n = 4$ simula-
 237 tions, a corresponding enhancement factor of $E = 1.8$ (simulation $n4E1.8$) is found.
 238 Henceforth, we refer to the $n3E0.5$ and the $n4E1.8$ simulations simply as the $n = 3$ and
 239 $n = 4$ simulations, respectively. Large fluctuations seen in Fig. 6 (c) and (d) are due
 240 to sudden changes in grounding line position and temporary localised decreases in el-
 241 evation. Simulations with an underestimated or overestimated enhancement factor re-
 242 sult in elevation and volumetric changes.

243 In both the $n = 3$ and the $n = 4$ simulation, there is a slight reduction in ice
 244 thickness at the divide compared with the BedMachine data of roughly 15 m (correspond-
 245 ing to 3 % of the ice thickness at the divide). This reduction in ice thickness is less in
 246 the $n = 4$ than the $n = 3$ simulation. On the western side of the ice rise excess thin-
 247 ning of the ice occurs, whereas in all other grounding zones, too much thickening occurs.
 248 Comparing the elevation change between the $n = 3$ and $n = 4$ simulations (Fig. 8), it
 249 can be seen that the steady-state $n = 3$ simulation has a lower elevation in the centre
 250 of the ice rise than the $n = 4$ simulation. In the north-eastern flank of the ice rise, the
 251 $n = 3$ simulation has a lower steady-state surface elevation than the $n = 4$ simula-
 252 tion, but all other flanks show a tendency for a higher surface elevation in the case of
 253 the $n = 3$ simulation.

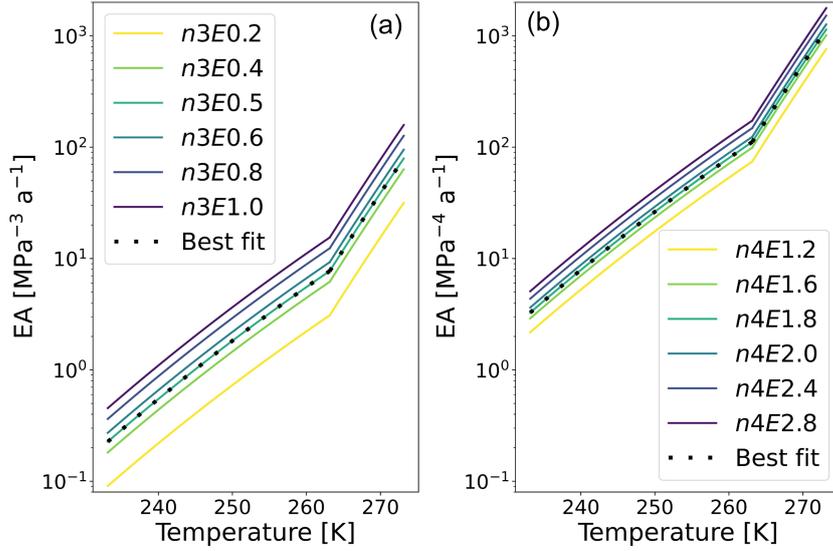


Figure 7. The relationship between the temperature and the Arrhenius law multiplied by varying enhancement factors, E ; (a) corresponds with the $n = 3$ simulations and (b) with the $n = 4$ simulations. Note the y -axes have a logarithmic scale. In (a) and (b), the dotted lines indicate the most suitable combination of parameters for the simulations of Derwael Ice Rise.

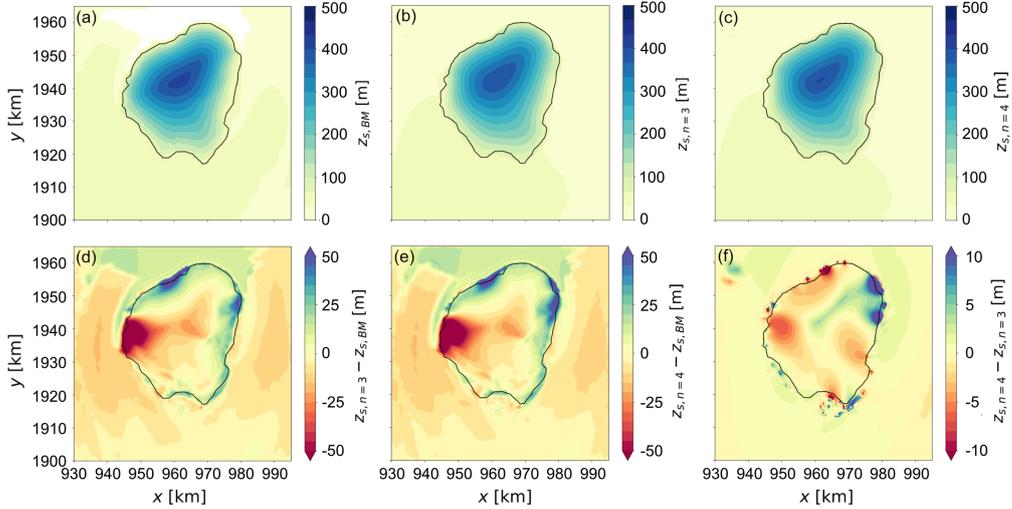


Figure 8. Surface elevation (a) from BedMachine Antarctica data, and (b) and (c) show the surface elevation of the $n = 3$ and $n = 4$ simulations, respectively, after the 400 year transient velocity spin up. The difference in surface elevation between the $n = 3$ simulation and the BedMachine data is shown in (d), the difference in surface elevation between the $n = 4$ simulation and the BedMachine data is shown in (e) and the difference in surface elevation between the $n = 4$ simulation and the $n = 3$ simulation is shown in (f). Note that (f) has a different colour scale than (d) and (e).

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4.2 Comparisons between modelled and observed stratigraphy

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Our new modelling framework allows us to further understand the similarities and differences between the observational data of Derwael Ice Rise and the simulated ice rise. Modelled isochrones are compared with dated isochrones derived from the radargrams obtained using airborne radar measurements. The radar measurements cover areas of Derwael Ice Rise including the ridge divide, flanks and grounding zones. The data is divided into seven cross-sections, each of which is compared with the model output for both $n = 3$ and $n = 4$ (Fig. 9). The modelled isochrones broadly reproduce the observed isochrones. The largest discrepancies between modelled and observed stratigraphy correspond to regions of the domain where the modelled and observed surface geometry do not match. For example, due to the tendency of the ice rise to broaden and thicken at the grounding zones and decrease in elevation in the centre during transient evolution, the elevation of the isochrones in these regions is generally under-estimated in the centre of the ice rise and over-estimated in the grounding zones. Evidence for this is provided by the cross-section $A-A'$ (Fig. 9a). There is a significant mismatch of up to 50 m between the modelled and observed isochrones, particularly on the eastern side which is likely due to an incorrect bed elevation. Mismatches with similar characteristics are also present in cross-sections $B-B'$, $C-C'$ and $F-F'$.

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Fig. 10 shows the difference between the observed and modelled $n = 3$ isochronal slope for the cross-sections $A-A'$ to $F-F'$ in Fig. 1. Generally, the slopes of the modelled isochrones match well with the observed isochronal slopes. Areas where there is a close match between observed and modelled isochrones are indicated by white in Fig. 10. At all grounding lines around the ice rise, there is significant steepening of isochrones due to the downward motion of ice. Given the general tendency of the ice rise to broaden in the grounding zones, the steepening of the modelled isochrones tend to be located a small distance from the observed isochrones, but reproduce similar patterns in isochrone geometry. On the stoss side of the ice rise, thinning of the stratigraphy indicates sudden acceleration of ice a few kilometres away from the grounding line. This is particularly evident in the cross-sections $A-A'$.

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In the observational data, the Raymond arch at the ridge divide is visible in the cross-sections $B-B'$, $C-C'$, $F-F'$ and $G-G'$. The side arch identified in Drews et al. (2015) is also visible in the cross-section $F-F'$. In Fig. 9e, the side arch visible in the observed isochrones is noticeable at a depth corresponding to the first modelled isochrone below the surface, which has an age of 100 years.

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4.3 Velocities and strain rates

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Simulated ice-shelf velocities on the western side of the ice rise are over 300 ma^{-1} and roughly 200 ma^{-1} on the eastern side. This is because of the location of the tributary Western Ragnhild Glacier. The asymmetry of the surrounding ice shelf results in an asymmetry in the divergence of the flow of ice around the ice rise. The simulated velocity field of the ice rise shows a distinct ridge divide on the northern side of the ice rise, with the divide positioned diagonally from the north east to the south west.

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Absolute surface velocity differences between the $n = 3$ and the $n = 4$ simulations are generally below 10 % with the exception of the divide and the north-eastern and south-eastern sides of the ice rise (Fig. ?? in the Supporting Information). The largest negative velocity differences occur in north-eastern and south-eastern sides of the ice rise with higher velocities in the $n = 3$ simulation. In the talwegs, velocities are higher in the $n = 4$ simulation. Note that talweg is a term first introduced in relation to ice rises in Gillet-Chaulet et al. (2011) and is a geomorphological term describing a valley. At the ridge divide, the velocities are significantly lower in the $n = 4$ simulation than the $n = 3$ simulation. In the areas perpendicular to the centre of the ridge divide, the $n = 4$ simulation has higher velocities than the $n = 3$ simulation. Elsewhere, the $n = 3$ sim-

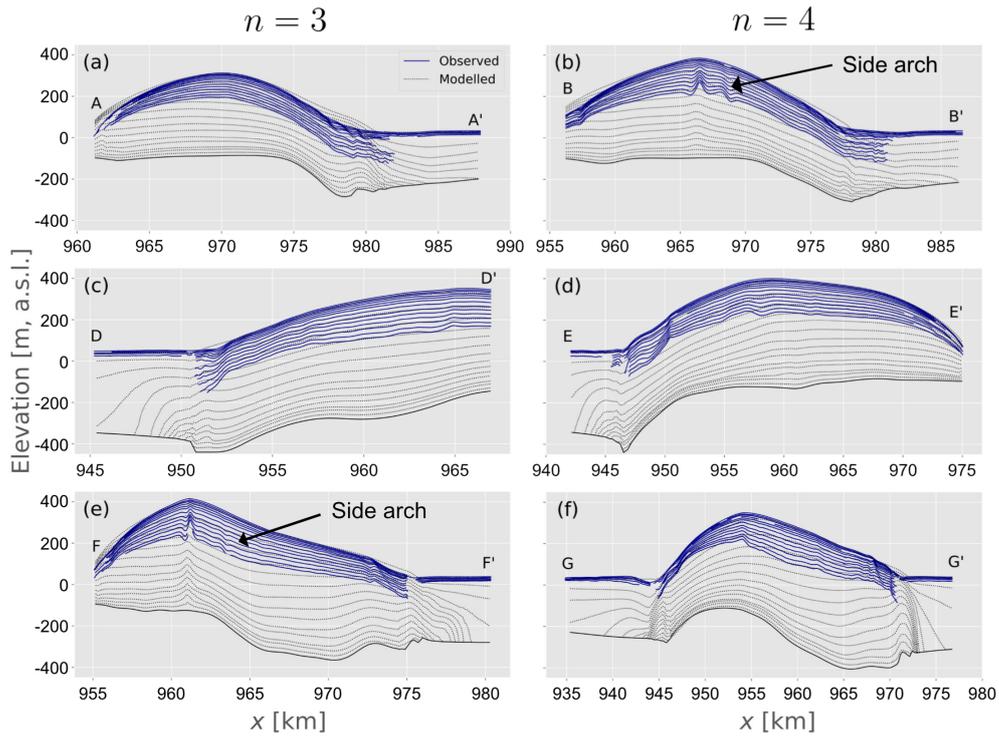


Figure 9. Comparisons between modelled and observed stratigraphy along radar profiles marked in 1 in the vertical domain of the model. Graphs in left columns, (a), (c), and (e), show comparisons for a Glen’s flow law exponent of $n = 3$ (cross-sections $A - A'$, $D - D'$ and $F - F'$), and the right column, (b), (d), and (f), show comparisons for a Glen’s flow law exponent of $n = 4$ (cross-sections $B - B'$, $E - E'$ and $G - G'$). The blue solid lines show the observed stratigraphy and the dotted black lines show the modelled stratigraphy.

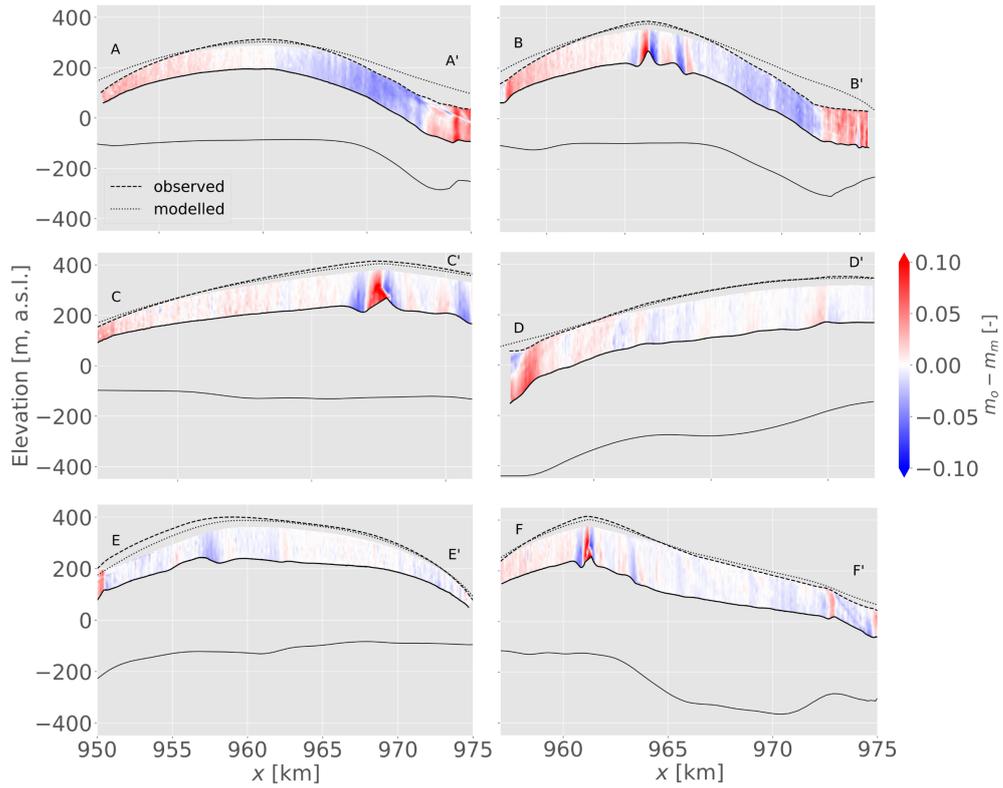


Figure 10. Difference between observed isochrone slope (m_o) and the $n = 3$ modelled isochrone slope (m_m) at locations where data is available for both. The cross-sections $A - A'$ to $F - F'$ correspond with the radar profiles in Fig. 1. The dashed lines show the observed ice surface and the dotted lines show the modelled surface. The lower extent of the area of comparison and the lower ice surface are shown with solid black lines.

305 ulation has higher velocities than the $n = 4$ simulation. A similar pattern of velocity
 306 difference between the $n = 3$ and the $n = 4$ simulations is observed at the depth of
 307 the 1000 year isochrone (Fig. ?? in the Supporting Information). Percentage differences
 308 in velocities are more pronounced in the talwegs at the 1000 year isochrone than at the
 309 ice surface. The flanks perpendicular to the ridge divide show higher velocities in the case
 310 of the $n = 4$ simulation than the $n = 3$ simulation. Furthermore, there are pronounced
 311 higher velocities in the $n = 3$ simulation in the south of the ice rise. Here, there is higher
 312 divergence of the velocity vectors (Fig. 12a), but not enough for an additional ridge or
 313 Raymond arches to form.

314 At the base of the ice rise, some basal sliding occurs. Lowest basal velocities of $<$
 315 1 ma^{-1} are simulated under the ridge divide and increase towards the flanks of the ice
 316 rise (Fig. ??a in the Supporting Information). Interestingly, from the centre of the ice
 317 rise to the south-eastern corner, there is an area of low velocity compared with elsewhere
 318 in the flanks of the ice rise. In three locations in the grounded ice, close to the ground-
 319 ing line, there is a higher basal sliding velocity of 5 ma^{-1} . This indicates that there is
 320 a higher effective stress in these areas, leading to acceleration of the ice. In both the $n =$
 321 3 and $n = 4$ simulations, the same basal friction parameterisation is used, and so differ-
 322 ences in the basal velocities are due to feedbacks with the overlying ice. A compar-
 323 ison of the basal velocities between the two simulations reveals that the largest differ-
 324 ences are seen in the grounding zones, where basal velocities in the $n = 4$ simulation
 325 are higher than in the $n = 3$ simulation (Fig. ??b in the Supporting Information). In
 326 the interior of the ice rise at the flanks of the ridge divide, velocities in the $n = 3$ sim-
 327 ulation are higher than in the $n = 4$ simulation.

328 The computed ice surface shear strain rate in the direction of ice flow shows a sim-
 329 ilar pattern to the ice velocity. Higher shear strain rates are observed on the western side
 330 of the ice rise. These result from the larger velocities in the ice shelf (Fig. 11). On the
 331 eastern side of the ice rise, the shear strain rates are lower than on the western side. The
 332 areas of higher shear strain rate on the eastern side are concentrated in two areas; in the
 333 north east and the south east of the ice rise. The area of lower shear strain rate between
 334 the areas of higher shear strain rate are a consequence of velocities from the ice rise and
 335 ice shelf being more similar in magnitude. Differences in shear strain rates between the
 336 $n = 3$ and $n = 4$ simulations primarily occur on the western side of the ice rise. Dif-
 337 ferences on the eastern side of the ice rise are negligible in comparison and localised dif-
 338 ferences are likely due to slight differences in grounding line position.

339 Differences in ice velocity between the simulations also affect the computed inter-
 340 nal stratigraphy. For both simulations, the oldest ice at a depth of 95 % is located at
 341 the ridge and on the stoss side of the ice rise. Here, convergence of flow from the ice rise
 342 and the ice shelf results in relatively stagnant ice velocities (Fig. ?? in the Supporting
 343 Information). The age field at a depth of 95 % shows that ice is on average 335 years
 344 older in the case of the $n = 3$ simulation. This reflects the higher strain rates and thus
 345 enhanced thinning of ice under a higher Glen's flow law exponent. At a depth of 50 %,
 346 differences in age are much less, with ice being between 25 and 50 years older at the di-
 347 vide in the case of the $n = 4$ simulation. The largest differences in age at a depth of
 348 50 % are seen in the area of compression between the ice rise and the ice shelf and in the
 349 north-eastern corner of the ice rise, with ice being more than 50 years older in the case
 350 of the $n = 4$ simulation. The opposite is seen in the talwegs, where ice is older in the
 351 case of the $n = 3$ simulation at a depth of 50 %.

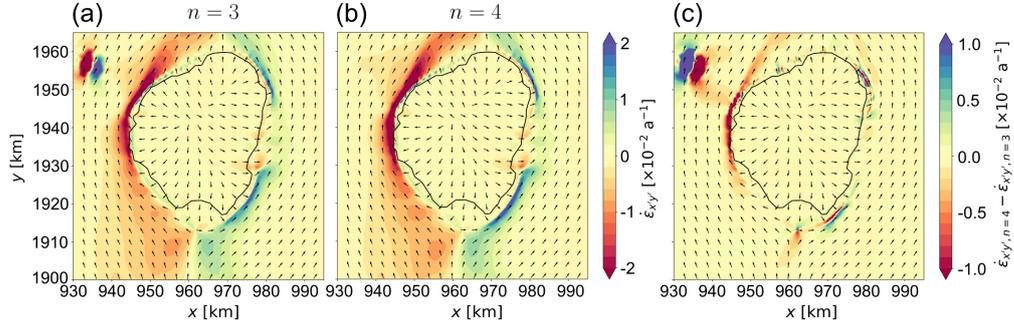


Figure 11. Shown in (a) and (b) is the shear strain rate for the $n = 3$ and $n = 4$ simulations, respectively, calculated by rotating the strain rate tensor to align with the velocity direction. In (c), the difference between the shear strain rate in the direction of the velocity of the $n = 4$ and $n = 3$ simulations is shown.

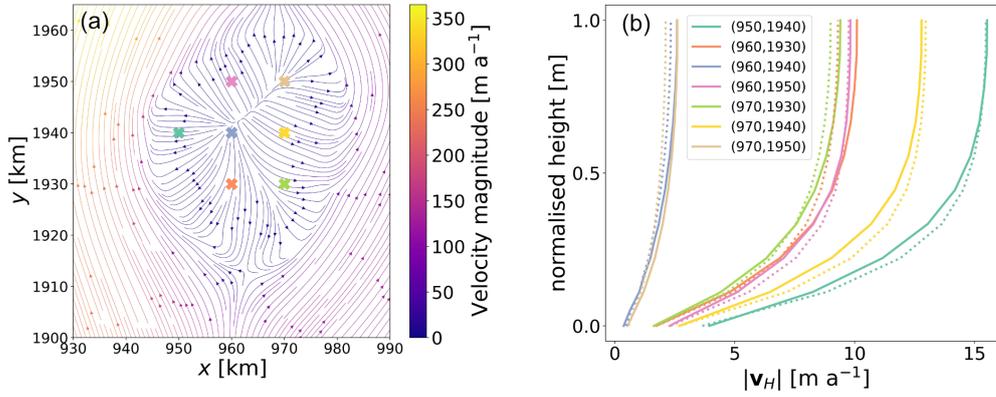


Figure 12. (a) shows the velocity field of Derwael Ice Rise, (b) shows the horizontal velocity magnitude for the (x, y) coordinates marked in (a), for the $n = 3$ simulation (solid lines) and the $n = 4$ simulation (dashed lines).

5 Discussion

5.1 Progress and challenges for three-dimensional ice rise modelling

Previous research has highlighted the importance of ice rises in deciphering past re-organisation of flow. Until now, the comparison between observed and modelled internal stratigraphy has been restricted to flow-line setups, providing a spatially limited assessment (Martín et al., 2006, 2009; Drews et al., 2015). We introduce a new three-dimensional modelling framework that overcomes these limitations and allows us to provide a spatially continuous age field that can be compared with radar observations. This provides an important step towards the routine simulation of ice rises and the ultimate goal of using them to constrain paleo ice-sheet simulations.

During the development of our modelling framework, we encountered a number of challenges. Given their small size, ice rises are often insufficiently resolved in continental-scale boundary datasets such as BedMachine or RACMO2.3p1. This can only be overcome if in-situ and high-resolution datasets are available to correct for these mismatches. The multiple steps necessary to spin-up the model are in parts subjective and a different sequence of spin-up steps of different simulation lengths may result in slightly differing results. As highlighted previously, it is important to note that interpolation errors in the bed elevation do occur and it may be necessary to make comparisons with the raw data. Failure to correct anomalies in the bed elevation data can result in artefacts in transient simulations, for example, a thickening of the ice rise after initialisation. This highlights the importance of such measurements to allow such studies for other ice rises around Antarctica.

The drift correction to the SMB implemented in our study results in an SMB field which is higher than the RACMO2.3p1 dataset (roughly 0.5 m a^{-1} higher) in the centre of the ice rise and lower closer to the margins of the ice rise than in the stratigraphy-adjusted SMB field. This points to a slight over-compensation of the ice softness and perhaps missing processes at the margins of the ice rise such as fracturing, higher melt rates or an anisotropic fabric or another process which would increase the velocity of the ice in that area. Another explanation is that the ice in the centre of the ice rise is stiffer in reality than in the model. As shown in Martín et al. (2009) and Martín and Gudmundsson (2012), anisotropic evolution of ice is a mechanism which enhances the stiffness of ice at an ice divide and the lack of this mechanism in our model is a likely reason for excess thinning of ice at the divide.

5.2 Comparison between modelled and observed stratigraphy

Much progress has been made in comparing modelled and observed internal stratigraphy (Sutter et al. (2021) and Born and Robinson (2021)) on large scales. For Stokes simulations, comparisons between modelled and observed stratigraphy have only been performed for two-dimensional simulations (Martín et al., 2009; Martín & Gudmundsson, 2012; Drews et al., 2015) and have so far not included the grounding line and the surrounding shelf. Including the ice shelf in the simulation domain means that grounding zone processes are included in the simulations and the domain boundaries are no longer within the bounds of the grounded area of the ice rise. Such a setup also allows investigation of isochrones in the shear zone between the ice rise and ice shelf which is characterised by steep isochrone geometries that are difficult to capture with radar observations.

The comparisons between the observed and modelled isochronal slopes in Fig. 10 show a close fit overall, with larger differences in the north east of the ice rise where issues with the bed elevation were found. Furthermore, differences are larger at the main Raymond arch where too coarse vertical resolution results in greater mismatches. Differences in isochrone slopes are primarily due to a mismatch in the surface elevation be-

402 tween modelled and observed results. The largest differences are seen in the north east
 403 of the ice rise and at the main Raymond arch and the side arch, which is not captured
 404 in the model. The side arch visible in the observed isochrones in Fig. 9e is visible at a
 405 depth corresponding to the first modelled isochrone below the surface which has an age
 406 of 100 years.

407 By studying areas where the surface elevation between the observed and modelled
 408 stratigraphy is similar, we can identify processes in ice dynamics which are not repro-
 409 duced by the model. In cross-section $D-D'$, there is a deviation in isochrone slope at
 410 $x = 957$ km when comparing the modelled isochrones to the observed isochrones. A
 411 greater thinning of the isochrones in the observed stratigraphy indicates that the mod-
 412 elled ice may not adequately reproduce speed-up of the ice in this area. In the cross-section
 413 $G-G'$, the acceleration of ice seen in the observed stratigraphy is not reproduced to
 414 the same extent in the model on the western side of the cross-section. On the eastern
 415 side of the $G-G'$ cross-section, the modelled stratigraphy shows more gentle slopes than
 416 the observed stratigraphy, indicating that in the direction of the grounding zone, mod-
 417 elled ice is accelerating more than the observed ice.

418 The side arch marked in Fig. 9b was discussed in previous work and it was suggested
 419 that it may be a result of unresolved three-dimensional effects (Drews et al., 2015). The
 420 lack of a side arch in our three-dimensional simulations indicates that this is not the case
 421 and we instead hypothesise that Derwael Ice Rise was previously a triple junction ice rise
 422 and that a ridge was previously located where the side arch is seen in the observed stratig-
 423 raphy. If this is the case, then a transition from a triple junction ice rise to a ridge di-
 424 vide ice rise is quite recent ($\lesssim 100$ years) as the side arches are also evident close to the
 425 surface. This then suggests that ice rises can have signatures of both ice-divide stabil-
 426 ity (evidenced by the oversized Raymond arches beneath the contemporary divide) and
 427 instability (evidenced by the side arch interpreted as a remnant of a ice-divide triple junc-
 428 tion). Furthermore, it cannot be ruled out that the flow divide had switched more than
 429 once between the main arch and the side arch.

430 Comparing the modelled isochrones to the observed isochrones in the shear zone,
 431 we see that in the grounding zone, the observed isochrones steepen closer to the ice rise
 432 interior than the modelled isochrones. This is due to a grounding line advance in the sim-
 433 ulations as a result of too little shear softening in the modelled shear zones around the
 434 ice rise, perhaps due to missing processes such as fracturing or an anisotropic fabric. An
 435 alternative approach to reducing the grounding line advance would be to alter the Ar-
 436 rhenius pre-factor to allow for softer ice, but this would result in a reduction of the ice
 437 rise elevation.

438 5.3 Glen's flow law exponent

439 In our study, we have investigated the influence of using a Glen's flow law expo-
 440 nent of $n = 3$ and $n = 4$. The $n = 4$ simulation results in a more peaked shape (Fig. 8).
 441 Both the $n = 3$ and the $n = 4$ simulations result in a slight lowering of the surface
 442 elevation in the ridge divide area, with the $n = 4$ simulation resulting in a surface el-
 443 evation closer to that of the observed surface elevation. This is an indicator that a Glen's
 444 flow law exponent of $n = 4$ is more suitable, but a general tendency for excess thicken-
 445 ing in the grounding zones means this result is not without uncertainty. The lower ve-
 446 locities in the $n = 4$ simulation compared with the $n = 3$ simulation align with our
 447 understanding of the Raymond effect, with lower velocities and greater Raymond arch
 448 amplitude associated with a higher Glen's flow law exponent. Interestingly, the differ-
 449 ences in the flanks of the ice rise indicate different responses to the non-linearity that
 450 result in slight differences in flow regime.

451 Predicted age–depth relationships differ by $< 5\%$ in most areas for simulations
 452 with $n = 3$ and $n = 4$ (Fig. ?? in the Supporting Information). This suggests that

453 tuning of the Arrhenius factor and the drift-correction for the $n = 3$ and $n = 4$ simu-
 454 lations lead to similar velocity fields in both cases. Exceptions are the divide regions and
 455 the shear zones at the ice-rise boundaries where differences are significant. Larger dif-
 456 ferences are seen in areas where stresses are significantly higher or lower than average
 457 stresses. Strain rates in the ice are higher in the $n = 4$ than the $n = 3$ simulation in
 458 the shear zones and near the ice-bed interface. Despite these differences, the modelled
 459 stratigraphy does not differ significantly. Studies for which an appropriate Arrhenius fac-
 460 tor for an exponent of $n = 4$ have so far only been performed with two-dimensional simu-
 461 lations (Martín et al., 2006; Drews et al., 2015), resulting in uncertainties due to the
 462 lack of through-plane velocities. A conversion of the Glen’s flow law from an exponent
 463 of $n = 3$ to $n = 4$ is further complicated by the dependence of the Arrhenius law on
 464 the temperature, activation energy, and n itself. Notably, significant uncertainty exists
 465 within these parameters as they have been calibrated through few laboratory studies in
 466 specific conditions (Zeitzi et al., 2020). In our conversion, we neglect differences in ac-
 467 tivation energies and use a typical stress, $[\tau_0]$, to calculate an initial guess for an appro-
 468 priate Arrhenius Law for $n = 4$. This simplified scaling is useful as we are able to com-
 469 pare parameters within our model to observed stratigraphy, highlighting the most ap-
 470 propriate values for the flow law. The chosen typical stress is within a reasonable range
 471 as in Goldsby and Kohlstedt (2001) and Goldsby (2006). In conjunction with known un-
 472 certainties in our understanding of the kinetics of glacier ice, implies that constraints on
 473 mechanisms such as temperature, activation energy, and grain size implicit within the
 474 Arrhenius relation are necessary to better understand the kinetics of creep on natural
 475 glacier ice.

476 Due to the assumption that ice is incompressible, the horizontal dilation (Fig. ??
 477 in the Supporting Information) is the equivalent of the vertical strain rate, $\dot{\epsilon}_{zz}$, with a
 478 sign change. We assume that differences in strain rate with and without a firn column
 479 do not differ greatly. The higher horizontal dilation in the north-western and eastern tal-
 480 wogs in the $n = 4$ simulation than the $n = 3$ simulation implies that there is greater
 481 stretching occurring in the ice. The opposite effect is seen in the vicinity of the ice rise
 482 ridge divide, with a lower dilation in the case of the $n = 4$ simulation. These small-scale
 483 results, which are in agreement with those presented in Gillet-Chaulet et al. (2011), are
 484 an analogy for larger scale situations. On larger scales, the higher dilation in areas of
 485 high velocity is likely to have consequences for the timing of the onset of an ice stream.
 486 Interestingly, in our simulations of Derwael Ice Rise, the south-western region of the ice
 487 rise shows a large area where the dilation is lower in the case of the $n = 4$ simulation
 488 than the $n = 3$ simulation, resulting in a region which does not contain a ridge divide,
 489 but also does not have strain rate differences which one would expect in the talweg of
 490 an ice rise. These characteristics are indicative of an ice rise which is in a state close to
 491 having a triple junction flow regime. Furthermore, the spatial variation in dilation and
 492 the resulting change in distance between isochrones could help in future to determine
 493 a correct Glen’s flow law exponent.

494 In Fig. 11, it can be seen that there are higher shear strain rates in the direction
 495 of flow in the shear margins in the case of the $n = 4$ simulation than in the $n = 3$ simu-
 496 lation. This result implies softer ice in shear margins when using a $n = 4$ simulation,
 497 indicative of viscous deformation at the higher strain rates. When investigating thresh-
 498 old shear strain rates or shear stresses beyond which fracturing occurs, it is important
 499 to bear in mind that a differing Glen’s flow law exponent will have a differing effect in
 500 simulations. An important observation is that the ice rumple north-west of Derwael Ice
 501 Rise becomes less grounded in the case of the $n = 4$ simulation, which we attribute to
 502 the greater strain-rate softening of the ice for the higher Glen’s flow law exponent. This
 503 has consequences for simulations including pinning points as the choice of Glen’s flow
 504 law exponent may have an influence on the buttressing due to that pinning point. More-
 505 over, the higher flux of ice into the ice shelf coming from the talweg on the eastern side

506 of Derwael Ice Rise, results in low shear strain rates in the grounding zone compared with
 507 the shear zones upstream and downstream.

508 **5.4 Model limitations and future research directions**

509 We have assumed that Derwael Ice Rise is in steady state and have found param-
 510 eter values which result in a steady-state geometry close to the present-day observed ge-
 511 ometry. Extra care would need to be taken when modelling other ice rises which do not
 512 satisfy the steady-state criterion. The boundary and initial conditions of our model are
 513 dependent on both observational data and model output (from regional atmospheric cli-
 514 mate model RACMO2.3p1). It is important to check for interpolation errors using the
 515 raw data. Failure to correct for the bed elevation led to a series of flawed transient sim-
 516 ulations in our case. Furthermore, we have not coupled anisotropy evolution to our model
 517 as there is insufficient anisotropy data available to constrain the model. Inclusion of ice-
 518 anisotropy will increase Raymond arch amplitudes. In future work, three-dimensional
 519 ice rises will provide ideal locations for the analysis of differing anisotropy schemes as
 520 well as other physical processes such as ice fracture in the shear margins.

521 Ice rises are good locations to study the effect of ice flow parameters across differ-
 522 ent flow regimes. The isochronal patterns observed near the base and surface are directly
 523 linked to the SMB and BMB fields. Simply adjusting the SMB and BMB fields using
 524 the change in surface elevation after initialisation does not, however, suffice for inferring
 525 the correct boundary conditions. We have therefore first adjusted A and n in the Glen's
 526 flow law (J. Glen, 1958). The parameter n has an influence on the proportions of the dome
 527 shape of the ice rise as shown in, for example, Gillet-Chaulet et al. (2011). A range of
 528 Arrhenius factors, A , then need to be tested with the various Glen's flow law exponents,
 529 n , in order to obtain an optimal ice rise geometry. A further source of uncertainty is in
 530 the basal friction parameterisation. Assuming that there is negligible basal sliding where
 531 there is substantial horizontal divergence of ice flow, an adjustment of the basal friction
 532 parameter can be made until there is sufficient thinning of ice in the talweg and no thin-
 533 ning elsewhere. We acknowledge that although we aim to independently adjust the ice
 534 flow parameterisations and boundary conditions they are none-the-less dependent on one-
 535 another. We argue, however, that with the steps we have taken in model calibration and
 536 comparison with isochrones, we have moved a step closer to independently determining
 537 model parameters.

538 **6 Conclusions**

539 We have introduced a new three-dimensional ice-rise modelling framework that in-
 540 cludes an ice rise, a grounding line, and the surrounding ice shelf. This framework al-
 541 lows us to compare the modelled three-dimensional stratigraphy with the observed stratig-
 542 raphy. The modelling framework presented here can be transferred as is to other ice rises
 543 of interest to predict the age-depth fields prior to ice-core drilling and also to continue
 544 constraining ice-flow parameters relevant for continent-wide simulations. Overall, we find
 545 that the modelled stratigraphy of Derwael Ice Rise matches well with observed stratig-
 546 raphy except in regions where there is uncertainty in the bed elevation. We predict 8000
 547 year old ice at 95 % depth and spatial age gradients at intermediate depth are signifi-
 548 cant reflecting the spatial variability in SMB. Observed arches in the ice-rise flanks can-
 549 not be reproduced and are likely a remnant of a former ice-divide triple junction that
 550 has disappeared in the last 100 years.

551 The presented modelling framework provides a blueprint for the simulation of other
 552 ice rises with Glen's flow exponents of $n = 3$ and $n = 4$ to make comparisons with ice
 553 cores or observational stratigraphy with the hopes of narrowing down uncertainties in
 554 other model parameters in the future, such as temperature and grain size. Simulations
 555 with differing $n = 3$ and $n = 4$ broadly result in similar velocity and age-depth fields

556 if the temperature dependent viscosity factors are tuned accordingly. Exceptions are ar-
 557 eas close to the ice divide, the peripheral shear zones and in the ice close to the ice-bed
 558 interface, helping to establish limits on the strain rates that permit viscous flow. Fur-
 559 thermore, this framework is a valuable first step towards testing and constraining var-
 560 ious physical processes such as fracturing and anisotropy, perhaps constrained with quad-
 561 polarimetric radar measurements (Ershadi et al., 2022).

562 Open Research Section

563 The code for the simulations can be found at [https://github.com/henryclara/](https://github.com/henryclara/Derwael/)
 564 [Derwael/](https://github.com/henryclara/Derwael/) and the code to produce the figures in the paper can be found at [https://](https://github.com/henryclara/DerwaelAccompanyingCode)
 565 github.com/henryclara/DerwaelAccompanyingCode. The data for producing the fig-
 566 ures and the data used as input to the model can be found at [https://nc-geophysik](https://nc-geophysik.guz.uni-tuebingen.de/index.php/s/7PdWiGeFJdFGMKH)
 567 [.guz.uni-tuebingen.de/index.php/s/7PdWiGeFJdFGMKH](https://nc-geophysik.guz.uni-tuebingen.de/index.php/s/7PdWiGeFJdFGMKH).

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