

# Supplementary Material of “Multiple Classifiers-Assisted Evolutionary Algorithm Based on Decomposition for High-Dimensional Multi-Objective Problems”

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This supplementary material supports the main manuscript entitled “Multiple Classifiers-Assisted Evolutionary Algorithm Based on Decomposition for High-Dimensional Multi-Objective Problems”. Specifically, it shows the complete IGD values for our sensitivity analysis of the population size  $N$  and a hyper-parameter  $R_{\max}$ .

## A. EXPERIMENTAL RESULTS WITH $N$

Tables I and II summarize the averaged IGD values on the DTLZ and WFG problems, respectively. Figs. 1 and 2 show the average ranks and the statistical results of the multiple test performed on the DTLZ and WFG problems, respectively. In addition, MOEA/D-EGO failed to solve DTLZ4 with  $D = \{100, 150\}$  because Kriging models fail to handle high-dimensional inputs.

## B. EXPERIMENTAL RESULTS WITH $R_{\max}$

Tables III and IV summarize the averaged IGD values of MCEA/D with  $R_{\max} = \{1, 10, 100, 1000\}$  on the DTLZ and WFG problems, respectively.  $R_{\max} = 10$  is used as a default value. In the table, “+”, “−”, and “ $\approx$ ” denote that the IGD value of MCEA/D with  $R_{\max} = \{1, 100, 1000\}$  is significantly better, worse, and competitive to that obtained by MCEA/D with  $R_{\max} = 10$ , respectively.

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TABLE I

AVERAGE IGD VALUES ON DTLZ PROBLEMS WITH  $N = 50$ . THE BEST IGD VALUE IS HIGHLIGHTED.(a)  $D = 50$ 

Prob.	$M$	EDN-ARMOEA	K-RVEA	MOEA/D-EGO	CPS-MOEA	CSEA	MCEA/D
DTLZ1	3	1.160e+03 –	1.122e+03 –	1.011e+03 –	8.967e+02 –	8.640e+02 –	5.772e+02
	7	8.318e+02 –	8.697e+02 –	7.549e+02 –	6.982e+02 –	6.006e+02 ≈	5.235e+02
	11	6.736e+02 –	7.401e+02 –	7.866e+02 –	5.962e+02 ≈	5.177e+02 ≈	5.337e+02
DTLZ2	3	2.912e+00 –	2.587e+00 –	2.316e+00 –	1.675e+00 –	1.819e+00 –	6.050e-01
	7	2.841e+00 –	2.659e+00 –	2.394e+00 –	2.118e+00 –	2.080e+00 –	9.654e-01
	11	2.745e+00 –	2.610e+00 –	2.796e+00 –	2.160e+00 –	2.107e+00 –	1.609e+00
DTLZ3	3	3.788e+03 –	3.512e+03 –	2.542e+03 –	2.680e+03 –	2.559e+03 –	1.164e+03
	7	3.256e+03 –	3.198e+03 –	1.560e+03 ≈	2.490e+03 –	2.353e+03 –	1.293e+03
	11	2.986e+03 –	3.030e+03 –	2.537e+03 –	2.178e+03 –	1.995e+03 –	1.158e+03
DTLZ4	3	2.998e+00 –	3.432e+00 –	2.488e+00 –	2.036e+00 –	1.701e+00 –	1.043e+00
	7	3.341e+00 –	3.286e+00 –	2.581e+00 –	2.450e+00 –	2.074e+00 –	1.262e+00
	11	3.118e+00 –	3.171e+00 –	2.382e+00 –	2.426e+00 –	1.937e+00 –	1.317e+00
DTLZ5	3	2.853e+00 –	2.372e+00 –	2.169e+00 –	1.625e+00 –	1.676e+00 –	4.312e-01
	7	2.545e+00 –	2.377e+00 –	2.008e+00 –	1.769e+00 –	1.741e+00 –	3.892e-01
	11	2.262e+00 –	2.144e+00 –	2.213e+00 –	1.564e+00 –	1.581e+00 –	9.040e-01
DTLZ6	3	4.063e+01 –	3.948e+01 –	2.189e+01 ≈	3.054e+01 –	4.130e+01 –	2.229e+01
	7	3.749e+01 –	3.690e+01 –	2.258e+01 ≈	2.895e+01 –	3.765e+01 –	2.137e+01
	11	3.385e+01 –	3.369e+01 –	2.859e+01 –	2.632e+01 –	3.401e+01 –	2.032e+01
DTLZ7	3	4.925e+00 +	8.279e+00 ≈	9.049e+00 ≈	9.112e+00 ≈	6.072e+00 +	9.188e+00
	7	1.208e+01 +	1.797e+01 +	2.167e+01 ≈	2.157e+01 ≈	2.497e+01 –	2.130e+01
	11	2.098e+01 +	2.493e+01 +	3.632e+01 –	3.385e+01 ≈	3.862e+01 –	3.320e+01
+ / – / ≈		3/18/0	2/18/1	0/16/5	0/17/4	1/18/2	-

(b)  $D = 100$ 

Prob.	$M$	EDN-ARMOEA	K-RVEA	MOEA/D-EGO	CPS-MOEA	CSEA	MCEA/D
DTLZ1	3	2.633e+03 –	2.824e+03 –	2.715e+03 –	2.009e+03 –	2.181e+03 –	1.311e+03
	7	1.876e+03 –	2.139e+03 –	2.016e+03 –	1.606e+03 –	1.712e+03 –	1.316e+03
	11	1.663e+03 –	1.898e+03 –	2.008e+03 –	1.423e+03 ≈	1.549e+03 –	1.338e+03
DTLZ2	3	6.603e+00 –	6.550e+00 –	5.839e+00 –	3.688e+00 –	4.802e+00 –	8.499e-01
	7	6.503e+00 –	6.490e+00 –	6.205e+00 –	4.023e+00 –	5.335e+00 –	1.159e+00
	11	6.318e+00 –	6.443e+00 –	6.459e+00 –	4.130e+00 –	4.944e+00 –	2.519e+00
DTLZ3	3	8.552e+03 –	8.935e+03 –	7.285e+03 –	5.795e+03 –	6.875e+03 –	3.013e+03
	7	8.096e+03 –	8.343e+03 –	5.383e+03 –	5.888e+03 –	6.817e+03 –	3.009e+03
	11	7.802e+03 –	8.003e+03 –	7.883e+03 –	5.752e+03 –	6.293e+03 –	3.251e+03
DTLZ4	3	6.704e+00 –	7.047e+00 –	-	3.888e+00 –	4.317e+00 –	1.218e+00
	7	6.953e+00 –	7.030e+00 –	-	4.214e+00 –	4.892e+00 –	1.386e+00
	11	6.731e+00 –	6.767e+00 –	-	4.373e+00 –	4.841e+00 –	1.420e+00
DTLZ5	3	6.407e+00 –	6.395e+00 –	5.756e+00 –	3.410e+00 –	4.750e+00 –	7.138e-01
	7	6.112e+00 –	6.309e+00 –	6.193e+00 –	3.756e+00 –	4.996e+00 –	6.444e-01
	11	5.774e+00 –	5.794e+00 –	5.984e+00 –	3.489e+00 –	4.452e+00 –	1.919e+00
DTLZ6	3	8.506e+01 –	8.644e+01 –	6.296e+01 –	6.514e+01 –	8.579e+01 –	5.052e+01
	7	8.109e+01 –	8.310e+01 –	6.346e+01 –	6.550e+01 –	8.242e+01 –	5.151e+01
	11	7.802e+01 –	7.968e+01 –	7.502e+01 –	6.301e+01 –	7.891e+01 –	4.739e+01
DTLZ7	3	6.798e+00 +	1.097e+01 –	1.009e+01 ≈	1.006e+01 ≈	7.136e+00 +	1.017e+01
	7	1.844e+01 +	2.374e+01 ≈	2.576e+01 –	2.424e+01 ≈	2.653e+01 –	2.403e+01
	11	3.061e+01 +	3.810e+01 ≈	4.126e+01 –	3.936e+01 ≈	4.207e+01 –	3.909e+01
+ / – / ≈		3/18/0	0/19/2	0/17/1	0/17/4	1/20/0	-

(c)  $D = 150$ 

Prob.	$M$	EDN-ARMOEA	K-RVEA	MOEA/D-EGO	CPS-MOEA	CSEA	MCEA/D
DTLZ1	3	4.135e+03 –	4.369e+03 –	4.279e+03 –	3.096e+03 –	3.468e+03 –	1.935e+03
	7	2.967e+03 –	3.402e+03 –	3.244e+03 –	2.510e+03 –	2.821e+03 –	1.987e+03
	11	2.651e+03 –	3.113e+03 –	3.225e+03 –	2.293e+03 –	2.611e+03 –	2.114e+03
DTLZ2	3	1.029e+01 –	1.037e+01 –	9.469e+00 –	5.122e+00 –	8.277e+00 –	1.067e+00
	7	1.021e+01 –	1.050e+01 –	1.003e+01 –	5.829e+00 –	8.680e+00 –	1.298e+00
	11	9.906e+00 –	1.031e+01 –	1.022e+01 –	6.107e+00 –	8.594e+00 –	3.433e+00
DTLZ3	3	1.340e+04 –	1.403e+04 –	1.206e+04 –	9.283e+03 –	1.146e+04 –	4.593e+03
	7	1.292e+04 –	1.354e+04 –	1.171e+04 –	9.430e+03 –	1.130e+04 –	4.438e+03
	11	1.250e+04 –	1.283e+04 –	1.268e+04 –	9.294e+03 –	1.112e+04 –	5.290e+03
DTLZ4	3	1.033e+01 –	1.074e+01 –	-	5.031e+00 –	7.172e+00 –	1.408e+00
	7	1.061e+01 –	1.072e+01 –	-	6.170e+00 –	8.195e+00 –	1.543e+00
	11	1.037e+01 –	1.055e+01 –	-	6.082e+00 –	7.942e+00 –	1.554e+00
DTLZ5	3	1.013e+01 –	1.039e+01 –	9.328e+00 –	5.190e+00 –	8.078e+00 –	9.830e-01
	7	9.858e+00 –	1.019e+01 –	9.772e+00 –	5.885e+00 –	8.356e+00 –	8.457e-01
	11	9.484e+00 –	9.787e+00 –	9.878e+00 –	5.525e+00 –	7.869e+00 –	2.749e+00
DTLZ6	3	1.295e+02 –	1.321e+02 –	9.904e+01 –	1.008e+02 –	1.305e+02 –	7.710e+01
	7	1.259e+02 –	1.282e+02 –	1.208e+02 –	1.026e+02 –	1.272e+02 –	7.676e+01
	11	1.224e+02 –	1.247e+02 –	1.191e+02 –	9.730e+01 –	1.233e+02 –	7.830e+01
DTLZ7	3	8.425e+00 +	1.128e+01 –	1.082e+01 ≈	1.042e+01 ≈	7.910e+00 +	1.058e+01
	7	2.136e+01 +	2.557e+01 ≈	2.682e+01 –	2.508e+01 ≈	2.731e+01 –	2.537e+01
	11	3.495e+01 +	4.071e+01 ≈	4.255e+01 –	4.090e+01 ≈	4.361e+01 –	3.985e+01
+ / – / ≈		3/18/0	0/19/2	0/17/1	0/18/3	1/20/0	-

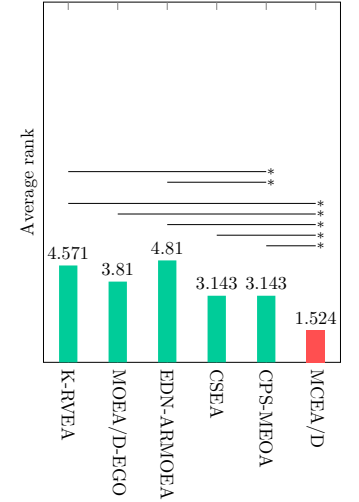
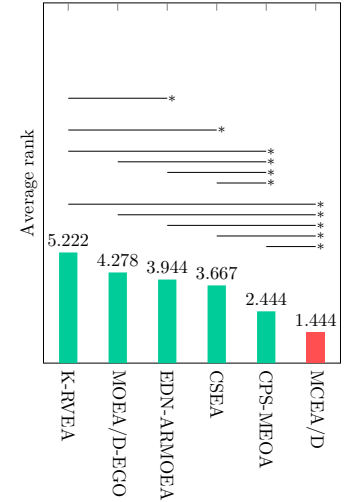
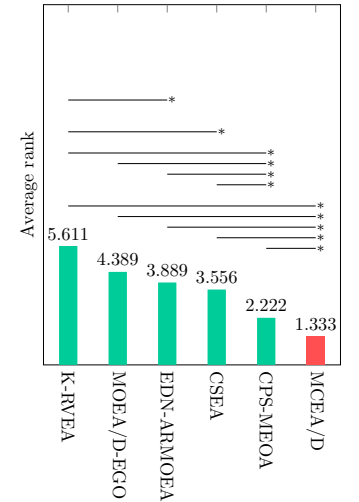
(a)  $D = 50$ (b)  $D = 100$ (c)  $D = 150$ Fig. 1. Average ranks and the multiple test results on DTLZ problems with  $N = 50$ .

TABLE II  
AVERAGE IGD VALUES ON WFG PROBLEMS WITH  $N = 50$ . THE BEST IGD VALUE IS HIGHLIGHTED.

(a)  $D = 50$

Prob.	$M$	EDN-ARMOEA	K-RVEA	MOEA/D-EGO	CPS-MOEA	CSEA	MCEA/D
WFG1	3	2.129e+00	2.331e+00	2.034e+00	2.261e+00	1.696e+00	2.227e+00
	7	2.839e+00	2.988e+00	2.835e+00	2.983e+00	2.786e+00	2.978e+00
	11	3.533e+00	3.628e+00	3.620e+00	3.661e+00	3.613e+00	3.688e+00
WFG2	3	8.670e-01	8.414e-01	8.939e-01	8.337e-01	6.972e-01	6.968e-01
	7	1.865e+00	2.478e+00	2.372e+00	1.994e+00	2.504e+00	2.035e+00
	11	3.312e+00	5.280e+00	4.845e+00	3.316e+00	4.231e+00	4.634e+00
WFG3	3	8.073e-01	7.917e-01	7.755e-01	7.699e-01	7.613e-01	5.766e-01
	7	1.413e+00	1.464e+00	1.456e+00	1.368e+00	1.322e+00	1.045e+00
	11	1.930e+00	1.997e+00	2.022e+00	1.837e+00	1.854e+00	1.360e+00
WFG4	3	5.620e-01	5.641e-01	6.271e-01	6.004e-01	5.389e-01	5.608e-01
	7	3.626e+00	3.091e+00	4.556e+00	2.994e+00	4.813e+00	3.180e+00
	11	9.643e+00	7.852e+00	1.217e+01	7.482e+00	1.207e+01	9.715e+00
WFG5	3	7.434e-01	7.635e-01	7.133e-01	6.283e-01	7.183e-01	5.784e-01
	7	3.366e+00	2.833e+00	3.842e+00	3.272e+00	3.873e+00	3.140e+00
	11	8.751e+00	6.060e+00	9.718e+00	8.217e+00	9.501e+00	8.918e+00
WFG6	3	9.406e-01	9.220e-01	1.015e+00	9.645e-01	9.022e-01	8.933e-01
	7	2.858e+00	3.026e+00	4.269e+00	3.373e+00	4.135e+00	3.718e+00
	11	7.237e+00	6.256e+00	1.093e+01	8.081e+00	1.005e+01	9.762e+00
WFG7	3	7.197e-01	7.223e-01	7.883e-01	7.228e-01	7.153e-01	6.374e-01
	7	2.709e+00	2.982e+00	4.297e+00	3.389e+00	4.281e+00	3.672e+00
	11	7.461e+00	6.401e+00	1.106e+01	8.838e+00	1.013e+01	1.062e+01
WFG8	3	7.908e-01	7.981e-01	9.246e-01	8.192e-01	7.940e-01	7.840e-01
	7	2.965e+00	3.024e+00	4.470e+00	3.366e+00	4.177e+00	3.804e+00
	11	7.445e+00	6.212e+00	1.065e+01	8.110e+00	1.015e+01	9.871e+00
WFG9	3	1.008e+00	1.018e+00	1.031e+00	9.168e-01	9.496e-01	7.571e-01
	7	4.011e+00	3.891e+00	4.539e+00	3.935e+00	4.412e+00	3.180e+00
	11	9.613e+00	9.581e+00	1.104e+01	9.061e+00	1.031e+01	9.085e+00
+ / - / $\approx$		10/11/6	10/10/7	3/2/2	11/12/4	4/15/8	-

(b)  $D = 100$

Prob.	$M$	EDN-ARMOEA	K-RVEA	MOEA/D-EGO	CPS-MOEA	CSEA	MCEA/D
WFG1	3	2.136e+00	2.424e+00	1.948e+00	2.279e+00	1.731e+00	2.228e+00
	7	2.853e+00	3.029e+00	2.843e+00	2.985e+00	2.749e+00	2.980e+00
	11	3.553e+00	3.692e+00	3.637e+00	3.667e+00	3.605e+00	3.679e+00
WFG2	3	8.624e-01	1.178e+00	9.270e-01	8.548e-01	7.417e-01	7.099e-01
	7	1.971e+00	3.382e+00	2.603e+00	1.908e+00	2.626e+00	1.890e+00
	11	3.387e+00	5.646e+00	5.224e+00	3.347e+00	4.778e+00	4.684e+00
WFG3	3	8.353e-01	8.740e-01	8.566e-01	8.054e-01	8.136e-01	5.820e-01
	7	1.443e+00	1.558e+00	1.519e+00	1.398e+00	1.445e+00	1.083e+00
	11	1.969e+00	2.075e+00	2.087e+00	1.918e+00	1.977e+00	1.389e+00
WFG4	3	5.999e-01	8.733e-01	6.479e-01	6.167e-01	5.505e-01	5.541e-01
	7	4.239e+00	5.428e+00	4.953e+00	2.967e+00	4.660e+00	3.175e+00
	11	1.074e+01	1.195e+01	1.202e+01	7.271e+00	1.233e+01	9.768e+00
WFG5	3	7.789e-01	9.316e-01	7.641e-01	6.501e-01	7.647e-01	5.992e-01
	7	3.567e+00	4.219e+00	4.109e+00	3.262e+00	3.885e+00	3.128e+00
	11	8.979e+00	1.008e+01	1.024e+01	8.278e+00	9.213e+00	8.632e+00
WFG6	3	9.624e-01	1.090e+00	1.039e+00	9.858e-01	9.471e-01	8.907e-01
	7	2.963e+00	4.626e+00	4.601e+00	3.305e+00	4.048e+00	3.744e+00
	11	7.579e+00	1.053e+01	1.069e+01	7.943e+00	9.688e+00	9.954e+00
WFG7	3	7.297e-01	9.233e-01	8.334e-01	7.435e-01	7.372e-01	6.353e-01
	7	2.865e+00	5.028e+00	4.792e+00	3.340e+00	3.962e+00	3.725e+00
	11	7.819e+00	1.092e+01	1.131e+01	8.424e+00	1.004e+01	1.061e+01
WFG8	3	7.942e-01	9.580e-01	9.444e-01	8.169e-01	8.091e-01	7.575e-01
	7	2.980e+00	4.726e+00	4.532e+00	3.279e+00	4.022e+00	3.770e+00
	11	7.734e+00	1.045e+01	1.088e+01	7.951e+00	9.918e+00	9.923e+00
WFG9	3	1.050e+00	1.195e+00	1.087e+00	1.005e+00	1.002e+00	7.765e-01
	7	4.053e+00	5.047e+00	4.640e+00	3.880e+00	4.394e+00	3.173e+00
	11	9.750e+00	1.096e+01	1.085e+01	9.042e+00	1.039e+01	9.046e+00
+ / - / $\approx$		10/16/1	0/25/2	3/23/1	10/13/4	4/18/5	-

(c)  $D = 150$

Prob.	$M$	EDN-ARMOEA	K-RVEA	MOEA/D-EGO	CPS-MOEA	CSEA	MCEA/D
WFG1	3	2.187e+00	2.428e+00	1.977e+00	2.276e+00	1.688e+00	2.226e+00
	7	2.881e+00	3.032e+00	2.868e+00	2.985e+00	2.811e+00	2.973e+00
	11	3.571e+00	3.705e+00	3.640e+00	3.671e+00	3.622e+00	3.690e+00
WFG2	3	8.763e-01	1.194e+00	9.683e-01	8.558e-01	7.792e-01	7.136e-01
	7	1.934e+00	3.358e+00	2.729e+00	1.989e+00	2.661e+00	1.864e+00
	11	3.567e+00	5.655e+00	5.520e+00	3.450e+00	4.765e+00	4.587e+00
WFG3	3	8.440e-01	8.898e-01	8.728e-01	8.133e-01	8.371e-01	5.887e-01
	7	1.456e+00	1.561e+00	1.557e+00	1.406e+00	1.478e+00	1.099e+00
	11	1.990e+00	2.098e+00	2.135e+00	1.913e+00	2.031e+00	1.428e+00
WFG4	3	6.187e-01	9.257e-01	6.676e-01	6.194e-01	5.597e-01	5.686e-01
	7	4.316e+00	5.645e+00	4.673e+00	2.982e+00	4.481e+00	3.187e+00
	11	1.101e+01	1.217e+01	1.244e+01	7.323e+00	1.234e+01	9.825e+00
WFG5	3	7.914e-01	9.416e-01	8.200e-01	6.564e-01	7.793e-01	6.289e-01
	7	3.550e+00	4.435e+00	4.241e+00	3.226e+00	3.905e+00	3.130e+00
	11	9.018e+00	1.015e+01	1.047e+01	8.258e+00	9.433e+00	8.618e+00
WFG6	3	9.747e-01	1.118e+00	1.062e+00	9.996e-01	9.710e-01	8.763e-01
	7	3.032e+00	4.846e+00	4.636e+00	3.251e+00	4.068e+00	3.657e+00
	11	7.834e+00	1.051e+01	1.067e+01	8.055e+00	9.585e+00	9.911e+00
WFG7	3	7.326e-01	9.632e-01	8.379e-01	7.447e-01	7.483e-01	6.447e-01
	7	2.904e+00	5.004e+00	4.713e+00	3.317e+00	4.078e+00	3.875e+00
	11	7.995e+00	1.077e+01	1.124e+01	8.344e+00	9.784e+00	1.022e+01
WFG8	3	7.898e-01	1.006e+00	9.604e-01	8.093e-01	8.174e-01	7.624e-01
	7	2.986e+00	4.779e+00	4.640e+00	3.275e+00	4.082e+00	3.691e+00
	11	7.727e+00	1.053e+01	1.074e+01	8.153e+00	9.962e+00	9.629e+00
WFG9	3	1.057e+00	1.247e+00	1.122e+00	1.024e+00	1.005e+00	7.431e-01
	7	4.047e+00	4.986e+00	4.654e+00	3.804e+00	4.393e+00	3.116e+00
	11	9.487e+00	1.085e+01	1.112e+01	9.061e+00	1.008e+01	8.773e+00
+ / - / $\approx$		10/16/1	0/27/0	3/24/0	11/13/3	3/18/6	-

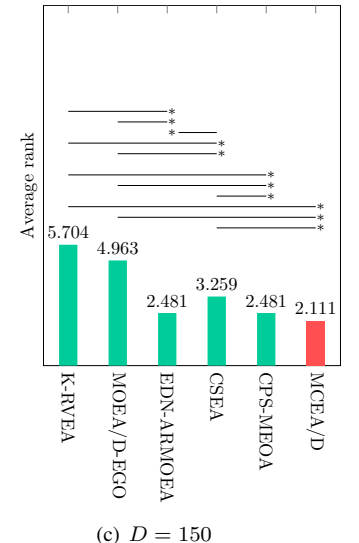
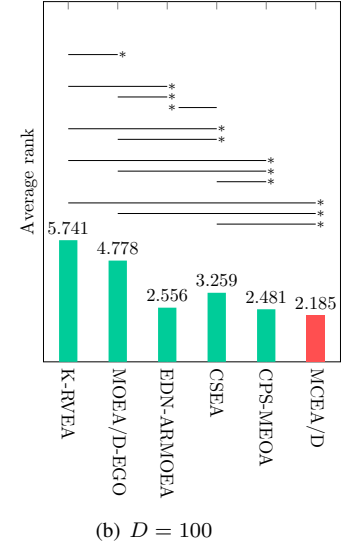
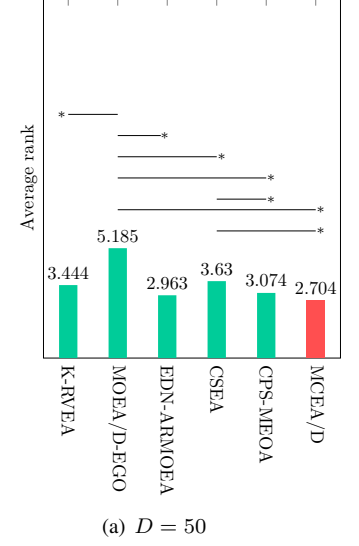


Fig. 2. Average ranks and the multiple test results on WFG problems with  $N = 50$ .

TABLE III

AVERAGE IGD VALUES ON DTLZ PROBLEMS WITH

 $R_{\max} = \{1, 10, 100, 1000\}$ . THE BEST IGD VALUE IS HIGHLIGHTED.(a)  $D = 50$ 

Prob.	$M$	$R_{\max} = 1$	$R_{\max} = 100$	$R_{\max} = 1000$	$R_{\max} = 10$
DTLZ1	3	7.864e+02 ≈	9.096e+02 −	9.962e+02 −	7.393e+02
	7	6.707e+02 ≈	6.810e+02 ≈	7.725e+02 −	6.963e+02
	11	5.568e+02 ≈	6.316e+02 −	6.406e+02 −	5.361e+02
DTLZ2	3	1.225e+00 −	5.946e-01 +	5.928e-01 +	6.789e-01
	7	1.732e+00 −	1.015e+00 +	9.989e-01 +	1.224e+00
	11	1.735e+00 −	1.260e+00 +	1.208e+00 +	1.442e+00
DTLZ3	3	1.997e+03 −	1.973e+03 −	1.962e+03 ≈	1.738e+03
	7	1.902e+03 −	1.923e+03 −	1.950e+03 −	1.540e+03
	11	1.635e+03 ≈	1.752e+03 ≈	1.767e+03 ≈	1.561e+03
DTLZ4	3	1.579e+00 −	1.059e+00 ≈	1.036e+00 +	1.075e+00
	7	1.716e+00 −	1.257e+00 +	1.263e+00 +	1.297e+00
	11	1.597e+00 −	1.311e+00 +	1.311e+00 +	1.333e+00
DTLZ5	3	1.159e+00 −	4.222e-01 +	4.130e-01 +	5.485e-01
	7	1.184e+00 −	4.235e-01 +	4.015e-01 +	6.087e-01
	11	1.183e+00 −	4.585e-01 +	4.200e-01 +	6.853e-01
DTLZ6	3	2.528e+01 ≈	2.591e+01 ≈	2.519e+01 ≈	2.505e+01
	7	2.526e+01 ≈	2.540e+01 ≈	2.534e+01 ≈	2.407e+01
	11	2.295e+01 −	2.429e+01 −	2.423e+01 −	2.128e+01
DTLZ7	3	9.212e+00 ≈	8.934e+00 ≈	9.086e+00 ≈	9.017e+00
	7	2.096e+01 ≈	2.107e+01 ≈	2.116e+01 ≈	2.123e+01
	11	3.500e+01 −	3.257e+01 ≈	3.348e+01 ≈	3.199e+01
+ / − / ≈		0/13/8	8/4/9	9/5/7	-

(b)  $D = 100$ 

Prob.	$M$	$R_{\max} = 1$	$R_{\max} = 100$	$R_{\max} = 1000$	$R_{\max} = 10$
DTLZ1	3	1.693e+03 ≈	2.067e+03 −	2.302e+03 −	1.705e+03
	7	1.578e+03 ≈	1.683e+03 ≈	1.734e+03 −	1.577e+03
	11	1.425e+03 ≈	1.572e+03 −	1.701e+03 −	1.394e+03
DTLZ2	3	2.325e+00 −	8.943e-01 +	8.504e-01 +	1.095e+00
	7	2.953e+00 −	1.216e+00 +	1.199e+00 +	1.912e+00
	11	2.889e+00 −	1.514e+00 +	1.429e+00 +	2.075e+00
DTLZ3	3	4.138e+03 ≈	5.431e+03 −	5.414e+03 −	3.790e+03
	7	4.410e+03 −	4.958e+03 −	4.924e+03 −	3.910e+03
	11	3.940e+03 ≈	4.568e+03 −	4.642e+03 −	3.889e+03
DTLZ4	3	2.433e+00 −	1.239e+00 +	1.228e+00 +	1.328e+00
	7	2.384e+00 −	1.412e+00 +	1.427e+00 +	1.517e+00
	11	2.167e+00 −	1.441e+00 +	1.438e+00 +	1.470e+00
DTLZ5	3	2.293e+00 −	7.520e-01 +	7.359e-01 +	1.023e+00
	7	2.550e+00 −	7.334e-01 +	6.580e-01 +	1.333e+00
	11	2.479e+00 −	7.414e-01 +	6.492e-01 +	1.529e+00
DTLZ6	3	5.750e+01 −	6.161e+01 −	6.189e+01 −	5.429e+01
	7	5.695e+01 ≈	5.935e+01 −	6.096e+01 −	5.440e+01
	11	5.393e+01 −	5.667e+01 −	5.674e+01 −	5.093e+01
DTLZ7	3	1.015e+01 ≈	1.050e+01 −	1.024e+01 ≈	9.991e+00
	7	2.469e+01 ≈	2.415e+01 ≈	2.392e+01 ≈	2.374e+01
	11	3.975e+01 ≈	4.026e+01 −	3.932e+01 ≈	3.844e+01
+ / − / ≈		0/12/9	9/10/2	9/9/3	-

(c)  $D = 150$ 

Prob.	$M$	$R_{\max} = 1$	$R_{\max} = 100$	$R_{\max} = 1000$	$R_{\max} = 10$
DTLZ1	3	2.629e+03 ≈	3.475e+03 −	3.388e+03 −	2.636e+03
	7	2.409e+03 ≈	2.728e+03 −	2.816e+03 −	2.376e+03
	11	2.243e+03 ≈	2.509e+03 −	2.595e+03 −	2.246e+03
DTLZ2	3	3.477e+00 −	1.172e+00 +	1.138e+00 +	1.489e+00
	7	3.990e+00 −	1.430e+00 +	1.333e+00 +	2.647e+00
	11	4.258e+00 −	1.795e+00 +	1.593e+00 +	2.985e+00
DTLZ3	3	6.477e+03 ≈	8.343e+03 −	8.613e+03 −	6.303e+03
	7	6.596e+03 ≈	7.903e+03 −	7.951e+03 −	6.114e+03
	11	6.734e+03 −	7.310e+03 −	8.291e+03 −	5.895e+03
DTLZ4	3	3.378e+00 −	1.466e+00 +	1.509e+00 ≈	1.556e+00
	7	3.492e+00 −	1.589e+00 +	1.637e+00 +	1.751e+00
	11	2.787e+00 −	1.598e+00 +	1.582e+00 +	1.637e+00
DTLZ5	3	3.040e+00 −	1.025e+00 +	1.040e+00 +	1.536e+00
	7	3.902e+00 −	9.098e-01 +	8.587e-01 +	2.149e+00
	11	3.517e+00 −	1.225e+00 +	8.659e-01 +	2.441e+00
DTLZ6	3	8.692e+01 ≈	9.638e+01 −	9.731e+01 −	8.780e+01
	7	8.860e+01 −	9.065e+01 −	9.774e+01 −	8.293e+01
	11	8.361e+01 ≈	8.651e+01 −	9.017e+01 −	8.175e+01
DTLZ7	3	1.028e+01 ≈	1.066e+01 ≈	1.060e+01 ≈	1.042e+01
	7	2.507e+01 ≈	2.516e+01 ≈	2.549e+01 ≈	2.531e+01
	11	4.098e+01 ≈	4.115e+01 −	4.097e+01 −	3.994e+01
+ / − / ≈		0/11/10	9/10/2	8/10/3	-

TABLE IV

AVERAGE IGD VALUES ON WFG PROBLEMS WITH

 $R_{\max} = \{1, 10, 100, 1000\}$ . THE BEST IGD VALUE IS HIGHLIGHTED.(a)  $D = 50$ 

Prob.	$M$	$R_{\max} = 1$	$R_{\max} = 100$	$R_{\max} = 1000$	$R_{\max} = 10$
WFG1	3	2.309e+00 ≈	2.248e+00 ≈	2.195e+00 +	2.283e+00
	7	3.006e+00 ≈	3.008e+00 ≈	2.997e+00 ≈	3.005e+00
	11	3.651e+00 ≈	3.669e+00 ≈	3.671e+00 ≈	3.659e+00
WFG2	3	7.863e-01 −	7.024e-01 ≈	7.191e-01 ≈	7.214e-01
	7	2.153e+00 ≈	2.470e+00 ≈	2.421e+00 ≈	2.181e+00
	11	3.873e+00 ≈	4.699e+00 ≈	4.787e+00 ≈	4.261e+00
WFG3	3	7.005e-01 −	5.353e-01 +	5.219e-01 +	5.811e-01
	7	1.223e+00 −	9.632e-01 +	9.507e-01 +	1.062e+00
	11	1.691e+00 −	1.267e+00 +	1.253e+00 +	1.357e+00
WFG4	3	5.897e-01 −	5.223e-01 ≈	5.057e-01 +	5.434e-01
	7	3.309e+00 ≈	4.092e+00 −	4.063e+00 −	3.534e+00
	11	7.887e+00 +	1.044e+01 −	1.097e+01 −	8.958e+00
WFG5	3	5.927e-01 +	6.575e-01 −	6.454e-01 −	6.184e-01
	7	3.477e+00 −	3.281e+00 ≈	3.283e+00 ≈	3.322e+00
	11	8.637e+00 −	8.227e+00 ≈	8.004e+00 +	8.317e+00
WFG6	3	8.890e-01 −	8.051e-01 +	8.080e-01 +	8.410e-01
	7	3.686e+00 ≈	3.895e+00 −	3.980e+00 −	3.672e+00
	11	8.653e+00 +	9.767e+00 −	9.851e+00 −	9.160e+00
WFG7	3	6.836e-01 −	6.113e-01 ≈	5.891e-01 +	6.252e-01
	7	3.881e+00 ≈	4.117e+00 ≈	4.129e+00 ≈	4.030e+00
	11	9.277e+00 +	1.063e+01 ≈	1.076e+01 −	1.026e+01
WFG8	3	8.228e-01 −	7.680e-01 ≈	7.530e-01 ≈	7.710e-01
	7	3.790e+00 ≈	4.017e+00 −	3.994e+00 −	3.758e+00
	11	8.622e+00 +	1.011e+01 −	1.007e+01 −	9.318e+00
WFG9	3	8.689e-01 −	6.996e-01 +	6.974e-01 +	7.746e-01
	7	3.983e+00 −	3.390e+00 +	3.293e+00 +	3.679e+00
	11	9.681e+00 −	8.159e+00 −	7.992e+00 +	9.059e+00
+ / − / ≈		5/13/9	7/7/13	11/8/8	-

(b)  $D = 100$ 

Prob.	$M$	$R_{\max} = 1$	$R_{\max} = 100$	$R_{\max} = 1000$	$R_{\max} = 10$
WFG1	3	2.315e+00 ≈	2.231e+00 ≈	2.223e+00 +	2.284e+00
	7	2.998e+00 ≈	2.999e+00 ≈	3.003e+00 ≈	2.995e+00
	11	3.650e+00 ≈	3.675e+00 ≈	3.678e+00 ≈	3.664e+00
WFG2	3	7.980e-01 −	7.617e-01 ≈	7.398e-01 ≈	7.260e-01
	7	2.138e+00 ≈	2.241e+00 ≈	2.404e+00 ≈	2.214e+00
	11	3.674e+00 ≈	4.864e+00 −	4.791e+00 ≈	4.109e+00
WFG3	3	6.941e-01 −	5.491e-01 +	5.462e-01 +	6.057e-01
	7	1.258e+00 −	9.891e-01 +	9.750e-01 +	1.100e+00
	11	1.675e+00 −	1.290e+00 +	1.270e+00 +	1.429e+00
WFG4	3	6.035e-01 −	5.307e-01 +	5.323e-01 +	5.569e-01
	7	3.294e+00 +	4.189e+00 −	4.377e+00 −	3.525e+00
	11	7.855e+00 +	1.071e+01 −	1.136e+01 −	9.140e+00
WFG5	3	6.133e-01 +	6.971e-01 −	7.239e-01 −	6.358e-01
	7	3.470e+00 −	3.270e+00 ≈	3.259e+00 ≈	3.341e+00
	11	8.864e+00 −	8.149e+00 +	7.800e+00 +	8.488e+00
WFG6	3	9.218e-01 −	8.441e-01 +	8.409e-01 +	8.831e-01
	7	3.675e+00 ≈	3.919e+00 ≈	4.039e+00 −	3.720e+00
	11	9.071e+00 ≈	9.756e+00 −	9.955e+00 −	9.289e+00
WFG7	3	6.916e-01 −	6.367e-01 ≈	6.319e-01 ≈	6.439e-01
	7	3.801e+00 ≈	4.223e+00 −	4.168e+00 −	3.862e+00
	11	9.351e+00 +	1.059e+01 −	1.068e+01 −	9.958e+00
WFG8	3	8.032e-01 −	7.613e-01 ≈	7.657e-01 ≈	7.584e-01
	7	3.744e+00 ≈	3.964e+00 −	4.019e+00 −	3.717e+00
	11	9.082e+00 ≈	9.819e+00 −	9.960e+00 −	9.402e+00
WFG9	3	9.084e-01 −	7.407e-01 +	7.404e-01 +	8.109e-01
	7	3.959e+00 −	3.254e+00 +	3.166e+00 +	3.623e+00
	11	9.468e+00 −	7.849e+00 +	7.562e+00 +	8.783e+00
+ / − / ≈		4/13/10	9/9/9	10/9/8	-

(c)  $D = 150$ 

Prob.	$M$	$R_{\max} = 1$	$R_{\max} = 100$	$R_{\max} = 1000$	$R_{\max} = 10$
WFG1	3	2.301e+00 $\approx$	2.172e+00 +	2.219e+00 $\approx$	2.269e+00
	7	2.996e+00 $\approx$	3.011e+00 $\approx$	2.998e+00 $\approx$	3.003e+00
	11	3.656e+00 $\approx$	3.673e+00 $\approx$	3.671e+00 $\approx$	3.663e+00
WFG2	3	7.880e-01 -	7.278e-01 $\approx$	7.498e-01 $\approx$	7.528e-01
	7	2.130e+00 $\approx$	2.315e+00 $\approx$	2.674e+00 -	2.117e+00
	11	3.846e+00 $\approx$	4.526e+00 -	4.729e+00 -	3.961e+00
WFG3	3	7.057e-01 -	5.643e-01 +	5.542e-01 +	6.116e-01
	7	1.298e+00 -	9.813e-01 +	9.661e-01 +	1.120e+00
	11	1.708e+00 -	1.284e+00 +	1.288e+00 +	1.436e+00
WFG4	3	6.010e-01 -	5.295e-01 +	5.417e-01 $\approx$	5.525e-01
	7	3.369e+00 $\approx$	4.452e+00 -	4.328e+00 -	3.488e+00
	11	8.069e+00 +	1.079e+01 -	1.103e+01 -	9.123e+00
WFG5	3	6.225e-01 -	7.200e-01 -	7.324e-01 -	6.346e-01
	7	3.452e+00 -	3.296e+00 $\approx$	3.243e+00 $\approx$	3.314e+00
	11	8.837e+00 -	8.078e+00 +	8.160e+00 +	8.444e+00
WFG6	3	3.178e-01 -	8.599e-01 $\approx$	8.647e-01 $\approx$	8.786e-01
	7	3.773e+00 $\approx$	3.955e+00 $\approx$	3.981e+00 -	3.761e+00
	11	1.198e+00 $\approx$	9.641e+00 $\approx$	9.924e+00 -	9.314e+00
WFG7	3	6.959e-01 -	6.431e-01 $\approx$	6.456e-01 $\approx$	6.507e-01
	7	3.753e+00 $\approx$	4.092e+00 -	4.177e+00 -	3.830e+00
	11	9.583e+00 $\approx$	1.039e+01 -	1.025e+01 -	9.620e+00
WFG8	3	7.803e-01 -	7.419e-01 $\approx$	7.500e-01 -	7.503e-01
	7	3.616e+00 $\approx$	3.881e+00 -	3.914e+00 -	3.662e+00
	11	8.889e+00 $\approx$	9.594e+00 $\approx$	9.976e+00 -	9.222e+00
WFG9	3	9.161e-01 -	7.858e-01 +	7.750e-01 +	8.176e-01
	7	3.916e+00 -	3.244e+00 +	3.115e+00 +	3.514e+00
	11	9.451e+00 -	7.833e+00 +	7.572e+00 +	8.683e+00
+ / - / $\approx$		1/13/13	97/11	71/19	-