# Reduction of nonlinear impairments in optical transmission systems for high-order quadrature-amplitude-modulation signals by using phase-conjugated twin waves

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**Abstract**

Using computer simulations, we analyze the bit-error rate (BER) characteristics of the optical transmission system repeated by erbium-doped fiber amplifiers (EDFAs) and two-mode phase-sensitive optical parametric amplifiers (PSAs), where phase-conjugated (PC) twin waves are co-propagating through the link. We focus on the 256 quadrature-amplitude-modulation (QAM) format, whose transmission characteristics are seriously affected by fiber nonlinearity because the required power is significantly high. Changing the repeater spacing and group-velocity dispersion (GVD) of fibers for transmission in the two-mode PSA-based system, we calculate the BERs as a function of the launched power. When the repeater spacing and GVD value of fibers are properly designed, the two-mode PSA-based system can mitigate nonlinear impairments more effectively than the EDFA-based PC twin system.

## 1 Introduction

The Kerr nonlinearity of optical fibers is a crucial consideration for designing long-distance coherent optical transmission systems [1]. When erbium-doped fiber amplifiers (EDFAs) were employed as repeaters in these systems, it was demonstrated that the use of phase-conjugated (PC) twin waves was effective for mitigating nonlinear impairments [2]. Given that self-phase modulation (SPM) of the signal wave is correlated with that of the co-propagating PC wave, the digital coherent receiver can significantly cancel out the SPM after receiving both the signal and PC waves.

Meanwhile, two-mode phase-sensitive optical parametric amplifiers (PSAs) were proposed in [3] and experimentally demonstrated in [4, 5, 6] for the purpose of reduction of their noise figure (NF) below 3 dB, which is the theoretical limit of EDFAs [7]. Also when two-mode PSAs are employed as repeaters, the PC wave is co-propagating through the link; consequently, SPM cancellation can be done at every repeater in an all-optical manner. Recent computer simulation results showed that the transmission characteristics of high-order quadrature-amplitude-modulation (QAM) signals could be improved by the two-mode PSAs owing to their low NF and all-optical nonlinearity-mitigation capability [8].

The present study extends our previous work [8] and analyzes the bit-error rate (BER) characteristics of the transmission system repeated by EDFAs and two-mode PSAs in detail, where the PC twins are co-propagating through the link. We focus on the 256 QAM format, the transmission characteristics of which are seriously affected by nonlinear impairments because the required power is significantly high. Changing the repeater spacing and group-velocity dispersion (GVD) of fibers, we calculate the

BER as a function of the launched power. The advantage of the transmission system using two-mode PSAs is clearly demonstrated, when the GVD and repeater spacing are properly designed.

## 2 Optical Transmission Systems Using PC Twin Waves

Figure 1 shows two kinds of transmission systems under study, both of which utilize PC twin waves for mitigation of nonlinear waveform distortion.

In Fig. 1 (a), EDFAs are employed as repeaters, and the GVD of fibers is accumulated along the entire system. In such a dispersion-unmanaged system, the accumulated GVD is compensated for at the digital coherent receiver (RX). The signal and complex conjugate of the PC wave are added together in the digital domain after the compensation for the accumulated GVD, resulting in SPM cancellation of the signal [2]. Wavelengths of the signal and PC waves are the same, whereas states of polarization of them are usually orthogonal.

In Figs. 1 (b), two-mode PSAs are employed as repeaters. In each PSA, the phase relation among the signal, PC, and pump waves are optimally controlled. The dispersion compensator (DC) compensates for the accumulated GVD of the fiber in every span, and this system is called the dispersion-managed system. Such dispersion compensation in the optical domain is indispensable for phase adjustment in each PSA. Note that SPM cancellation is done at each PSA in an all optical manner. Finally, the signal is demodulated by RX. The signal and PC waves are not degenerated in the wavelength domain, whereas states of polarization of them are usually the same.

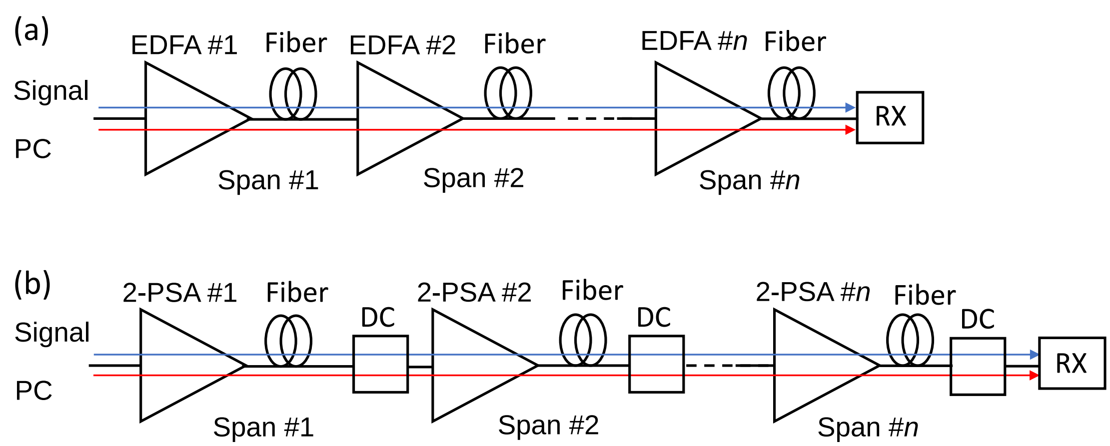


Figure 1: Optical transmission systems using PC twin waves for fiber-nonlinearity mitigation. In Fig. (a), EDFA-based repeaters are employed for transmission. The accumulated GVD is compensated for at the digital coherent receiver (RX). In Fig. (b), repeaters based on two-mode PSAs are employed for transmission. DCs denote optical dispersion compensators. The GVD of the fiber is compensated for in every span using the DC.

## 3 Gain and Noise Characteristics of EDFs and Two-Mode PSAs

This section summarizes the gain and noise characteristics of EDFAs and two-mode PSAs, which are used in computer simulations in Sec. 5 and Sec. 6. The role of the PC twins is demonstrated, and the formula for the amplifier noise is also introduced.

### 3.1 EDFAs

The gross gain of an EDFA, $G_0$, is written as

$$G_0 = \exp(gt) \ , \tag{1}$$

where $g$ denotes the gain coefficient and $t$ is the travelling time of the signal in the EDFA. Using the loss coefficient, $\gamma$, the internal loss of the EDFA, $\Gamma_0$, is given as

$$\Gamma_0 = \exp(-\gamma t) \ . \tag{2}$$

Then, the net gain is written as

$$G = G_0 \Gamma_0 \ . \tag{3}$$

The real and imaginary parts of the signal electric field are amplified by the same gain of $\sqrt{G}$.

The power of amplified spontaneous emission (ASE) from the EDFA per polarization is written as

$$P_n = \frac{hf(G-1)n_{sp}}{T_s} \ , \tag{4}$$

where $hf$ denotes the photon energy, $n_{sp}$ is the spontaneous emission factor, and $T_s$ represents the measurement time interval [9] [1]. In Eq. (4), the spontaneous emission factor, $n_{sp}$, is given as

$$n_{sp} = \frac{g}{g-\gamma} \ . \tag{5}$$

The real and imaginary parts of the amplified signal electric field have the same noise power of $P_n/2$. Therefore, in computer simulations, two independent sets of Gaussian noise having the variance of $P_n/2$ are added to the real and imaginary parts of the amplified electric field. Moreover, when PC twin waves, which belong to orthogonal modes, are amplified simultaneously, they suffer from independent noise.

The signal and PC waves are amplified by an EDFA and combined by a digital coherent receiver. The average of the receiver output is written as

$$\langle a_s \rangle = \frac{\sqrt{G}}{2} \left( \langle a_{s,0} \rangle + \left\langle a_{i,0}^* \right\rangle \right) \ , \tag{6}$$

where $\langle a_{s,0} \rangle$ and $\langle a_{i,0} \rangle$ stand for the average inputs of the signal and PC-wave electric fields, respectively. At the input, we prepare the PC wave satisfying

$$\langle a_{i,0} \rangle = \left\langle a_{s,0}^* \right\rangle \ . \tag{7}$$

Then, we have

$$\langle a_s \rangle = \sqrt{G} \langle a_{s,0} \rangle \ . \tag{8}$$

Meanwhile, the noise power in the output, $a_s$, is given by

$$P_n = \frac{hf\,(G-1)\,n_{sp}}{2T_s} \ , \tag{9}$$

because the ASE noise power associated with each of the amplified signal and PC wave is written as Eq. (4). Compared to Eq. (4), the noise power is halved; on the other hand, Eq. (6) shows that the equivalent input power is given by (signal power + PC power)/4. Therefore, the NF of the amplifier is unchanged even by using PC twin waves, and is given by [9]

$$\mathrm{NF} = 2n_{sp} \ , \tag{10}$$

which can approach 3 dB [2].

[1] The corresponding optical filter bandwidth is given by $\Delta f = 1/Ts$. In computer simulations, $T_s$ means the sampling time interval. In other words, $1/T_s$ represents the sampling rate.

[2] Note that $n_{sp} \geq 1$.

### 3.2 Two-Mode PSAs

In the case of a two-mode PSA, the average signal electric field, $\langle a_s \rangle$, is expressed as [10]

$$\langle a_s \rangle = \frac{1}{2}\left[\left(\sqrt{G_+} + \sqrt{G_-}\right)\langle a_{s,0}\rangle + \left(\sqrt{G_+} - \sqrt{G_-}\right)\langle a_{i,0}^*\rangle\right] \ , \tag{11}$$

where $\langle a_{s,0}\rangle$ and $\langle a_{i,0}\rangle$ stand for the average inputs of the signal and idler electric fields, respectively. The phase-sensitive net gains, $G_+$ and $G_-$, are given by

$$G_+ = \left(\sqrt{G_0} + \sqrt{G_0 - 1}\right)^2 \Gamma_0 \ , \tag{12}$$

$$G_- = \left(\sqrt{G_0} - \sqrt{G_0 - 1}\right)^2 \Gamma_0 \ . \tag{13}$$

The gross gain of optical parametric amplifiers, $G_0$, is expressed as

$$G_0 = \cosh^2\left(\frac{gt}{2}\right) \ . \tag{14}$$

where $g$ denotes the gain coefficient and $t$ is the traveling time of the signal in the PSA. The internal loss, $\Gamma_0$, is also given by Eq. (2). Preparing the idler input satisfying Eq. (7), we find that Eq. (11) yields the signal output expressed as

$$\langle a_s \rangle = \sqrt{G_+}\,\langle a_{s,0}\rangle \ . \tag{15}$$

When $G_+ \gg G_-$, Eq. (11) is approximated as Eq. (6). It means that SPM cancellation can be done in an all-optical manner in the two-mode PSA. Moreover, ASE associated with the phase sensitive gain, $G_+$, dominantly contributes to the amplifier noise. Consequently, the ASE power is given by [10]

$$P_n = \frac{hf\,(G_+ - 1)\,n_{sp}}{4T_s} \ . \tag{16}$$

The spontaneous emission factor, $n_{sp}$, is the same as Eq. (5). The real and imaginary parts of the amplified electric field have the same gain of $\sqrt{G_+}$ and the same ASE power of $P_n/2$. Using the value of $P_n$, computer simulations are carried out by the method applied to EDFAs.

Equation (16) shows that the noise power in the signal is half of Eq. (9) if $G = G_+$. Therefore, the NF of the two-mode PSA is 3-dB lower than that of the EDFA, and is given by [10]

$$\mathrm{NF} = n_{sp} \ , \tag{17}$$

which can approach 0 dB. Thus, noise-free amplification is achieved in such a case.

# 4 Nonlinear Propagation Through Fibers

Two-mode signal propagation through a fiber is governed by the nonlinear Schrödinger equation, given by

$$\frac{\partial E_{s,i}}{\partial z} = -\frac{\alpha}{2}E_{s,i} - \frac{i}{2}\beta_2\frac{\partial^2 E_{s,i}}{\partial T^2} + \frac{8i}{9}\gamma_n\left(|E_s|^2 + |E_i|^2\right)E_{s,i}, \tag{18}$$

where $E_s$ denote the signal electric field, and $E_i$ is the electric field of the PC or idler wave, $\alpha$ is the loss coefficient, $\beta_2$ is the GVD parameter, and $\gamma_n$ represents the nonlinear coefficient. Instead of using $\beta_2$, the GVD value of fibers is also expressed by

$$D = -\frac{2\pi c\beta_2}{\lambda^2} \ , \tag{19}$$

where $c$ is the light velocity, and $\lambda$ is the wavelength. We neglect four wave mixing in Eq. (18), which occurs when the state of polarization of $E_s$ is aligned to that of $E_i$.

Equation (18) shows that the SPM term for $E_s$ and $E_i$ is the same. Therefore, taking $E_s + E_i^*$ at the receiving end, we can cancel the SPM considerably. However, dispersive waveform distortion for $E_s$ is different from that for $E_i$, because $E_s$ and its phase conjugate experience signs of the GVD opposite to each other. This results in the imperfect SPM cancellation.

Using Eq. (18) and the optical-amplifier model described in Sec. 3, we perform computer simulations of the QAM transmission characteristics in Sec. 5 and Sec. 6 [1].

## 5 Transmission Characteristics of 256 QAM Signals

Figure 2 shows computer simulation results of the BER characteristics of 65-Gbaud 256 QAM signals transmitted through a 500-km link, in which the repeater spacing is fixed at 50 km. In the EDFA-based system shown by Fig. 1(a), standard single-mode fibers (SMFs) are employed for transmission along the entire system. The GVD value of the SMFs is $D$ = 17 ps/nm/km at $\lambda$ =1.55 $\mu$m, the loss coefficient, $\alpha$, is 0.2 dB/km, and the nonlinear coefficient, $\gamma_n$, is 1.5 /km/W. The spontaneous emission factor, $n_{sp}$, is 1.58 (=2 dB) [3].

In the PSA-based system shown by Fig. 1(b), dispersion-shifted fibers (DSFs) are employed in addition to the SMFs. Given that the GVD value of the DSFs is zero, DCs are not inserted in the link. Meanwhile, when the SMFs are used in the dispersion-managed link, the DCs compensate for the accumulated GVD in each span, but we ignore the loss of the DCs. The loss coefficient and nonlinear coefficient of the DSFs are 0.2 dB/km and 2.6 /km/W, respectively. The spontaneous emission factor, $n_{sp}$, of the PSAs is 1.58 (=2 dB) [4].

The red curve shows the BERs calculated as a function of the total launched power, that is, the sum of the signal and PC-wave powers measured at the amplifier output, when PC twin waves and EDFA-based repeaters are used. For reference, the green curve demonstrates the BERs of the conventional EDFA-based system, where the PC wave is not incident. We find that nonlinear impairments can be mitigated significantly by the use of the PC twin waves.

The blue curve shows the BERs of the PSA-based system, where the SMFs are used in the dispersion-managed link. Compared with the red BER curve, the receiver sensitivity of this system is improved by 3 dB owing to the lower NF of the PSAs. However, its nonlinearity-mitigation capability is almost the same as that of the EDFA-based PC-twin system (red curve). The black BER curve is obtained in the dispersion-free link, where the DSFs with the zero GVD value are employed. In this system, nonlinear impairments are drastically reduced. Dispersive waveform distortions for $E_s$ and $E_i$ are so small that the SPM is cancelled effectively.

Thus, we find that the nonlinearity-mitigation capability of the two-mode PSA-based system is strongly dependent on the GVD value of fibers in the dispersion-managed link. In Sec. 6, we concentrate on studying the effect of the GVD value.

## 6 Effect of GVD value of Fibers for Transmission on Efficient Nonlinearity Mitigation by Two-Mode PSAs

Figure 3 shows the BERs calculated when the GVD value and repeater spacing are varied in the two-mode PSA-based system. The total transmission distance is 500 km. In Figs. (a), (b), (c), and (d), the repeater spacing is 50 km, 25 km, 12.5 km, and 6.25 km, respectively. Conversely, the black, blue, red, brown, and green curves are obtained when $D$ = 0, -1, -2, -5, and -10 ps/nm/km, respectively [5].

[3]The NF of the EDFAs is 5 dB.

[4]The NF of the two-mode PSAs is 2 dB.

[5]The negative $D$ is called the normal dispersion. Compared to the anomalous dispersion, which means the positive $D$, the normal dispersion reveals somewhat better BER characteristics.

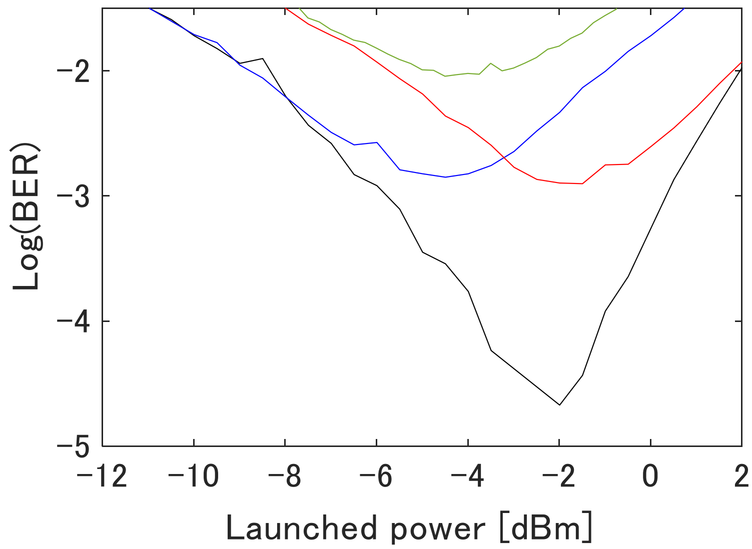


Figure 2: BER characteristics of the 256 QAM signal. The transmission distance and repeater spacing are 500 km and 50 km, respectively. The red curve corresponds to the BERs obtained in the system shown by Fig. 1 (a), where SMFs are used for transmission. For reference, the green curve shows the BERs of the conventional EDFA-based system, where the PC wave is not incident. The blue curve demonstrates the BERs obtained in the system shown by Fig. 1 (b), where SMFs are used for transmission. Meanwhile, the black BER curve is obtained in the system shown by Fig. 1 (b), where DSFs are used for transmission.

We assume that $\gamma_n = 2.6$ /km/W in all cases.

We find that the shorter repeater spacing brings forth the better BER characteristics. The impact of shortening the repeater spacing is two fold: First, the launched power required to maintain a certain BER is reduced, and hence, the nonlinear impairments are diminished. Second, all-optical SPM cancellation is performed more effectively, because dispersive waveform distortion becomes smaller owing to smaller accumulated GVD. From theses reasons, the power range, where the BER is kept below a certain value (for example, $10^{-5}$), is extended in both of the high and low power sides. On the other hand, reducing the GVD value, we find that SPM cancellation is achieved more effectively. This is also because of smaller dispersive waveform distortion.

In conclusion, the shorter repeater spacing and smaller GVD value can improve the all-optical SPM cancellation performance, resulting in the better BER characteristics.

# 7 Conclusion

Through intensive computer simulations, we have analyzed the BER characteristics of a 256 QAM transmission system repeated by EDFAs and two-mode PSAs, where the PC twins are co-propagating through the link. It has been demonstrated that the system using two-mode PSAs can outperform that using EDFAs, provided that the GVD value of fibers for transmission and repeater spacing are properly designed. In such a system, SPM is cancelled out effectively at every PSA-based repeater in an all-optical manner. The low NF of PSAs is also helpful for enhancing the BER performance.

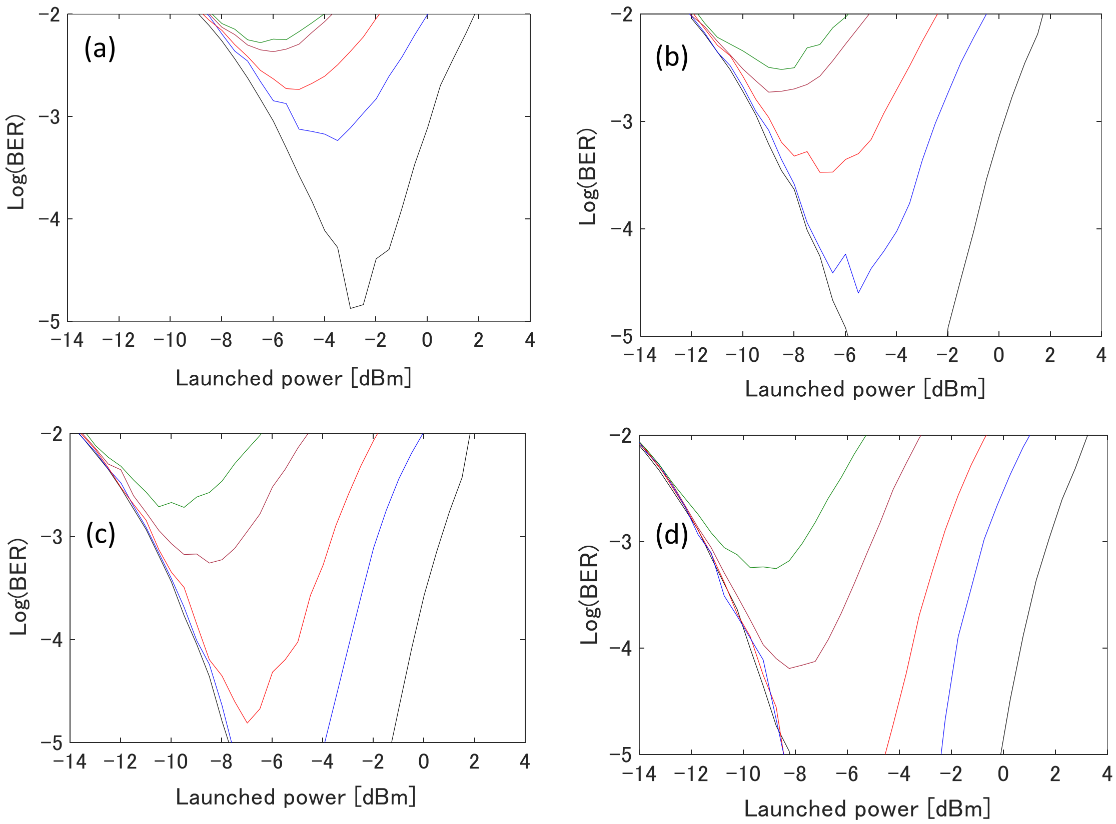


Figure 3: BER characteristics of the 256 QAM signal transmitted through the two-mode PSA-based system. The transmission distance is 500 km. In Figs. (a), (b), (c), and (d), the repeater spacing is 50 km, 25 km, 12.5 km, and 6.25 km, respectively. The black, blue, red, brown, and green curves are obtained when $D = 0$, -1, -2, -5, and -10 ps/nm/km, respectively. We assume that $\gamma_n = 2.6$ /km/W in all cases.

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