

Computational Dualism and Objective Superintelligence

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Abstract. The concept of intelligent software is flawed. The behaviour of software depends upon the hardware that interprets it. This undermines claims regarding the behaviour of theorised, software superintelligence. Here we characterise this problem as “computational dualism”, where instead of mental and physical substance, we have software and hardware. We argue that to make objective claims regarding performance we must avoid computational dualism. We propose using an alternative based upon pancomputationalism, wherein every aspect of the environment is a relation between irreducible states. We formalise systems as behaviour (inputs and outputs), and cognition as embodied, embedded, extended and enactive. The result is cognition formalised as a part of the environment, rather than as a disembodied policy interacting with the environment through an interpreter. This allows us to make objective claims regarding intelligence, which we argue is the ability to “generalise”, identify causes and adapt. We then propose objective upper bounds for intelligent behaviour.

Keywords: enactivism · computational dualism · AGI · AI safety.

1 Introduction

AIXI [1] is a general reinforcement learning agent. It uses Solomonoff Induction, a formalism of Ockham’s Razor [2], to make accurate inferences from minimal data. It was initially thought to be pareto optimal, representing an upper bound on intelligence [3]. Unfortunately, this claim was later shown to be a matter of interpretation [4]. We argue that this is a valuable insight indicative of a much larger problem with how artificial intelligence (AI) is conceived. We call this problem “computational dualism”, in reference to René Descartes’s interactionist, substance dualism. We then discuss an alternative formulation of intelligent behaviour that permits objective performance claims. It is based upon enactive cognition [5], pancomputationalism [6] and weak constraint optimisation [7]. We use it to propose upper bounds on intelligent behaviour.

Intelligence: Our focus is not AIXI, but intelligence. General intelligence is often defined in terms of predictive models [1, 8, 9]. A more “intelligent” predictive

model *generalises*, making accurate predictions in *unfamiliar* circumstances. It *adapts*. The more efficiently one adapts, the more “intelligent” one is. Predictive accuracy is not what matters. With enough training examples even a lookup table can make accurate predictions (because it will have seen every example). What matters is how *efficiently* one learns what *caused* examples. Knowing cause, one can make accurate predictions. Intuitively, a model *explains* the present by identifying those aspects of the past which caused it. Using such a model, we might use the more distant past to explain events in the more recent past, and the present to predict the future. It is the future with which we are concerned, as an agent that can accurately predict the results of its actions can choose the actions that yield the most reward. Hence the ability to adapt to unforeseen circumstances, satisfy goals and otherwise behave intelligently can be equated with the ability to identify cause and effect [10]. Of course, the only *truly* accurate model of the environment *is* the environment. Everything else is an abstraction. Hence even if we only consider models that comprehensively explain the past, some will “generalise” better than others.

Simplicity: This is where Ockham’s Razor comes in. All else being equal, it implies that simpler models are more likely to generalise [11]. Simplicity is often formalised as Kolmogorov Complexity (KC) [12] or minimum description length [13]. The KC of an object is the length of the shortest self extracting archive of that object. Intuitively, the KC of the past is the shortest comprehensive description of the past. More compressed descriptions tend to generalise better. This is why some believe that compression and intelligence are closely related [14]. Formally, in the case of AIXI, if the model which generated past data is indeed computable, then the simplest model will dominate the Bayesian posterior as more and more data is observed. Eventually, AIXI will have identified the correct model, which it can use to generate the next sample (predict the future).

Subjectivity: We will now use AIXI to illustrate the problem with software “intelligence”. KC (and thus AIXI’s performance) is measured in the context of a UTM. By itself, changing the UTM would not meaningfully affect performance. When used in a universal prior to predict deterministic binary sequences, the number of incorrect predictions a model will make is bounded by a multiple of the KC of that model [2]. If the UTM is changed the number of errors only changes by a constant [15, pp. 2.1.1 & 3.1.1], so changing the UTM doesn’t change which model is considered most plausible. However, when AIXI employs this prior in an *interactive* setting, a problem occurs [4]. Intuitively (with significant abuse of notation), assume a program f_1 is software, f_2 is an interpreter and f_3 is the reality (an environment, body etc) within which goals are pursued. According to Ockham’s Razor, AIXI is the optimal choice of f_1 to maximise the performance of $f_3(f_2(f_1))$. However, in an interactive setting one’s perception of success may not match reality.

“Legg-Hutter intelligence [3] is measured with respect to a fixed UTM. AIXI is the most intelligent policy if it uses the same UTM.” [4, p.10]

Using our informal analogy of functions, this means performance in terms of $f_3(f_2(f_1))$ depends upon $f_2(f_1)$, not f_1 alone. A claim regarding the performance of f_1 alone would be *subjective*, in that it depends upon f_2 .

“This undermines all existing optimality properties for AIXI.” [4, p.1]

Computational dualism: We suggest this problem has broader significance for AI. The concept of a software “mind” interacting with a hardware “body” echoes Descartes’ interactionist, substance dualism [16]. Descartes argued mental and physical “substances” interact through the pineal gland which interprets mental events to cause physical events [17], like a UTM interprets software. When later scholars pointed out the inconsistencies implied by this interaction between mental and physical, some argued that the mental “supervenes” on the physical, meaning any two objects that are exactly the same mentally must be exactly the same physically. More recently, philosophers have proposed purely physicalist depictions of the mind [18], and have even gone so far as to formalise cognition not just as embodied, but as enacted through the environment [5]. One might argue that AI is the engineering branch of philosophy of mind and cognitive science. Instead of mental and physical substances, we have software and hardware. Software “supervenes” on hardware; any two computers that are exactly alike in hardware configuration down to the value of each and every bit, must be exactly alike in terms of software. However, we have not moved on from dualism and formalised more recent conceptions of mind. AI still tends to be equated with “immortal” software. Unless there exists a Platonic realm of pure math (akin to a mental realm), Hinton’s “immortal” computations [19] do not exist and never have. There are only embodied “mortal” computations, because software is nothing more than the configuration of hardware. Our understanding of AI should be revised to reflect this. This is what we set out to do here.

Summary: We propose computational dualism, argue AI warrants an alternative, discuss one such alternative¹, and then propose an objective upper bound on intelligent behaviour.

2 Formalising Enactivism

To make objective claims we must avoid computational dualism. One alternative is enactivism [24]. It holds that mind, body and environment are inseparable. Cognition extends into the environment, and is enacted through what the organism does. For example, if someone uses pen and paper to solve a math problem, then cognition is enacted using the pen and paper [25]. To formalise enactivism, we must formalise cognition as a part of the environment. We do so in pancomputationalist [6] terms, albeit a very minimal interpretation thereof. Pancomputationalism holds that everything is a computational system. Using the analogy from earlier, we formalise $f_2(f_3(f_1))$ instead of $f_3(f_2(f_1))$. One may

¹ Definitions or variations thereof are shared with related work [7, 20, 21, 22, 23].

regard the interpreter f_2 as the laws of nature, the environment f_3 as software running on f_2 , and we're seeking to portray the mind f_1 as subject to the environment. Because of this, the distinction between software and hardware must be discarded. Instead, we describe artificial minds in a purely behaviourist manner [9]. We describe inputs and outputs rather than a mechanism by which one is mapped to the other. Correct policies are then all possible “causal intermediaries” between a set of inputs and outputs, akin to functionalist explanations of human mentality [18].

The environment: Rather than assuming programs, we arrive at a pancomputational model of the environment by assuming first that some things exist, some do not, and that the environment is that which exists. Second, we assume there is at least one state of the environment. States could be differentiated along dimensions like time, but we don't strictly need to assume that. We also don't need to assume anything about the internal structure or contents of states. Instead, we can formalise the set of all declarative programs² as the powerset of states. A declarative program is “true” about every state it contains, and false about everything else. Every conceivable environment or state thereof amounts to a set of true declarative programs, or “facts”.

Definition 1 (environment).

- We assume an infinite set Φ whose elements we call **states**.
- A **declarative program** is $f \subseteq \Phi$, and we write P for the set of all declarative programs (the powerset of Φ).
- By a **truth** or **fact** about a state ϕ , we mean $f \in P$ such that $\phi \in f$.
- By an **aspect of a state** ϕ we mean a set l of facts about ϕ s.t. $\phi \in \bigcap l$. By an **aspect of the environment** we mean an aspect l of any state, s.t. $\bigcap l \neq \emptyset$. We say an aspect of the environment is **realised**³ by state ϕ if it is an aspect of ϕ .

Our approach reflects Goertzel's framing of unary, dyadic and triadic relations [26]. A state here is unary, referring only to itself. However, a declarative program is dyadic relation between states, in the sense that it refers to states which are its truth conditions. Sets of declarative programs form a lattice based on truth conditions, which relates them to one another. We can then define triadic relations by taking three sets of declarative programs i, o, π such that π is a “causal intermediary” implying o given i .

Embodiment: We use these dyadic relations to formalise enactivism [5], holding that everything we call the mind is just part of the environment, and “thinking” is just changes in environmental state. This blurs the line between agent and environment, making the distinction unclear. As Heidegger maintained, Being is bound by context [27]. There is no need to define an agent that has no environment, and so there seems to be little point in preserving the distinction.

² Intuitively, declarative programs are anything which is true or false.

³ Realised meaning it is made real, or brought into existence.

What we need is not a Cartesian model of disembodied intelligence looking in upon an environment but an embodiment, embedded in a particular part of the environment, through which goal directed behaviour can be enacted. We need to describe the *circuitry* with which cognition is embodied and enacted, much like a **formal language** only with truth conditions determined by environmental states. Just as every computational system we can build is finite, and living systems are ergodic [28], we assume all of this takes place within a system which has a finite number of configurations, and so amounts to a finite number of declarative programs. We call this an **abstraction layer**. Intuitively, an abstraction layer is like a window. It looks onto that part of the environment in which cognition takes place. For example, we might enumerate every possible declarative program which pertains only to one specific computer, and use that computer as our abstraction layer. However, as we can take any set of declarative programs and define an abstraction layer, it is “pancomputational” in the sense that it captures every system in computational terms.

Definition 2 (abstraction layer).

- We single out a subset $\mathfrak{v} \subseteq P$ which we call **the vocabulary** of an abstraction layer. The vocabulary is finite unless explicitly stated otherwise. If $\mathfrak{v} = P$, then we say that there is no abstraction.
- $L_{\mathfrak{v}} = \{l \subseteq \mathfrak{v} : \bigcap l \neq \emptyset\}$ is a set of aspects in \mathfrak{v} . We call $L_{\mathfrak{v}}$ a formal language, and $l \in L_{\mathfrak{v}}$ a **statement**.
- We say a statement is **true** given a state iff it is an aspect realised by that state.
- A **completion** of a statement x is a statement y which is a superset of x . If y is true, then x is true.
- The **extension of a statement** $x \in L_{\mathfrak{v}}$ is $E_x = \{y \in L_{\mathfrak{v}} : x \subseteq y\}$. E_x is the set of all completions of x .
- The **extension of a set of statements** $X \subseteq L_{\mathfrak{v}}$ is $E_X = \bigcup_{x \in X} E_x$.
- We say x and y are **equivalent** iff $E_x = E_y$.

(notation) E with a subscript is the extension of the subscript⁴.

Intuitively, $L_{\mathfrak{v}}$ is everything which can be realised in this abstraction layer. The extension E_x of a statement x is the set of all statements whose existence implies x , and so it is like a truth table. Now that we have an embodiment, we need to define goal directed behaviour. An abstraction layer is *already* goal directed in the sense that it constrains what might be described, just as living organisms evolve to thrive in particular environments. However, an organism can then engage in more *specific* goal directed behaviours by *learning* and adapting to remain fit in more circumstances. This is what we seek to formalise *using* an abstraction layer. It is where a predictive model might fit. However we do not need a value-neutral model of the environment [9].

“The best model of the world is the world itself.” - Rodney Brooks [29]

⁴ e.g. E_l is the extension of l .

The only aspects of the environment that we might actually need model are those necessary to satisfy goals [30]. What we need is a model of a task, or more specifically to enumerate the goal directed behaviour (inputs and outputs) that will eventually cause the task to be completed. Goal directed here just means some outputs are “correct”, and some are not. It is arbitrary.

Definition 3 (v-task). For a chosen \mathbf{v} , a task α is a pair $\langle I_\alpha, O_\alpha \rangle$ where:

- $I_\alpha \subset L_{\mathbf{v}}$ is a set whose elements we call **inputs** of α .
- $O_\alpha \subset E_{I_\alpha}$ is a set whose elements we call **correct outputs** of α .

I_α has the extension E_{I_α} we call **outputs**, and O_α are outputs deemed correct. $\Gamma_{\mathbf{v}}$ is the set of **all tasks** given \mathbf{v} .

(generational hierarchy) A \mathbf{v} -task α is a **child** of \mathbf{v} -task ω if $I_\alpha \subset I_\omega$ and $O_\alpha \subseteq O_\omega$. This is written as $\alpha \sqsubset \omega$. If $\alpha \sqsubset \omega$ then ω is then a **parent** of α . \sqsubset implies a “lattice” or generational hierarchy of tasks. Formally, the level of a task α in this hierarchy is the largest k such there is a sequence $\langle \alpha_0, \alpha_1, \dots, \alpha_k \rangle$ of k \mathbf{v} -tasks where $\alpha_0 = \alpha$ and $\alpha_i \sqsubset \alpha_{i+1}$ for all $i \in (0, k)$. A child is always “lower level” than its parents.

(notation) If $\omega \in \Gamma_{\mathbf{v}}$, then we will use subscript ω to signify parts of ω , meaning one should assume $\omega = \langle I_\omega, O_\omega \rangle$ even if that isn’t written.

Intuitively, an **input** is a possibly incomplete description or task. An **output** is a completion of an input [def. 2]. We treat **correctness** as binary. An output is correct if it causes the task to become complete to some acceptable degree with some acceptable probability⁵. Degrees of complete or correct just reflect different \mathbf{v} -task definitions⁶. For example we might put this in evolutionary terms, where the inputs and outputs are the enumeration of all “fit” behaviour in which an organism might engage, as remaining alive is goal directed behaviour. We now formalise learning and inference of goal directed behaviour as tasks. Inference requires a “policy”. Being a set of declarative programs, a correct policy *is* the goal of a \mathbf{v} -task, so this amounts to goal learning.

Definition 4 (inference).

- A \mathbf{v} -task **policy** is a statement $\pi \in L_{\mathbf{v}}$. It constrains how we complete inputs.
- π is a **correct policy** iff the correct outputs O_α of α are exactly the completions π' of π such that π' is also a completion of an input.
- The set of all correct policies for a task α is denoted Π_α .⁷

Assume \mathbf{v} -task ω and a policy $\pi \in L_{\mathbf{v}}$. Inference proceeds as follows:

1. we are presented with an input $i \in I_\omega$, and

⁵ Also called “satisficing” a goal [31].

⁶ Affect or reward and attribution thereof are beyond this paper’s scope.

⁷ To repeat the definition in set builder notation: $\Pi_\alpha = \{\pi \in L_{\mathbf{v}} : E_{I_\alpha} \cap E_\pi = O_\alpha\}$

2. we must select an output $e \in E_i \cap E_\pi$.
3. If $e \in O_\omega$, then e is correct and the task “complete”. $\pi \in \Pi_\omega$ implies $e \in O_\omega$, but $e \in O_\omega$ doesn’t imply $\pi \in \Pi_\omega$ (an incorrect policy can imply a correct output).

Intuitively, a **policy** constrains how we complete inputs. It is a **correct policy** if it constrains us to correct outputs. To “learn” a policy we use a **proxy**. A proxy estimates one thing, by measuring another seemingly unrelated thing. For example, AIXI uses simplicity to estimate model veracity. In our case, we want a policy that classifies correct outputs. We will use a proxy called “weakness”, which has been shown to outperform simplicity in sample efficiency and causal learning [7, 20]. Where simplicity is a property of *form*, weakness is a property of *function*. In order to make objective claims regarding performance, we cannot rely upon subjective interpretations of form.

Definition 5 (learning).

- A **proxy** $<$ is a binary relation on statements. $<_w$ is the **weakness** proxy. For statements l_1, l_2 we have $l_1 <_w l_2$ iff $|E_{l_1}| < |E_{l_2}|$.

(generalisation) A statement l **generalises** to a **v-task** α iff $l \in \Pi_\alpha$. We speak of **learning** ω from α iff, given a proxy $<$, $\pi \in \Pi_\alpha$ maximises $<$ relative to all other policies in Π_α , and $\pi \in \Pi_\omega$.

(probability of generalisation) We assume a uniform distribution over Γ_v . If l_1 and l_2 are policies, we say it is less probable that l_1 generalizes than that l_2 generalizes, written $l_1 <_g l_2$, iff, when a task α is chosen at random from Γ_v (using a uniform distribution) then the probability that l_1 generalizes to α is less than the probability that l_2 generalizes to α .

(sample efficiency) Suppose **app** is the set of **all pairs of policies**. Assume a proxy $<$ returns 1 iff true, else 0. Proxy $<_a$ is more sample efficient than $<_b$ iff

$$\left(\sum_{(l_1, l_2) \in \text{app}} |(l_1 <_g l_2) - (l_1 <_a l_2)| - |(l_1 <_g l_2) - (l_1 <_b l_2)| \right) < 0$$

(optimal proxy) There is no proxy more sample efficient than $<_w$, so we call $<_w$ **optimal**. This formalises the idea that “explanations should be no more specific than necessary” (see Bennett’s razor in [7]).

Learning is an activity undertaken by some manner of agent, and a task has been “learned” when that agent knows a correct policy. For example, one has “learned” chess when one knows the rules and some winning strategies. Here instead of agents, we have embodied goal-directed behaviours in the form of **v-tasks**. Humans typically learn from “examples”. In the context of a **v-task** an “example” is a correct output and an input that is a subset thereof. A collection of examples is a child task. “Learning” is an attempt to generalise from a known child to one of its parents. Intuitively, a child functions like a “history” or memory

of *correct* interactions (an “ostensive definition”). The lower the child is in the generational hierarchy one learns from, the more sample efficiently one learns. We assume tasks are **uniformly distributed** because anything else would imply that environment “prefers” or makes a value judgement about goals. The environment *is* a value judgement about what exists, but is otherwise assumed to be impartial. Consequently the most sample efficient proxy is $<_w$ [7]. It enables causal learning [20] as the weakest correct policies have the highest probability of being the causal intermediary between inputs and outputs.

3 Limits of Intelligence

As stated earlier, intelligence is often understood in terms of “the ability to generalise” [8] and learn the policy which “caused” examples [20]. We wish to understand the *objective* upper limits of intelligence. $<_w$ is the optimal choice of proxy for sample efficiency [7, prop. 1, 2, 3]. In the context of a fixed abstraction layer, the upper bound of intelligent behaviour is attained by the weakness proxy. However, more intelligence is not always useful. Intuitively, making a human more intelligent is unlikely to improve their driving. Likewise, intelligence conveys no advantage if one’s purpose is to ascribe meaning to random noise. The extent to which it is *possible* to construct weak correct policies depends on both the *abstraction layer* and the task. An abstraction layer is a bottleneck on intelligence, and can be goal directed just like a task. We can measure this goal directed “utility” as the extent to which it is *possible* to construct weak correct policies given a task in a particular abstraction layer.

Definition 6 (utility of intelligence). $\epsilon : \Gamma_{\mathbf{v}} \rightarrow \mathbb{N}$ is a function that takes a \mathbf{v} -task γ and returns a “utility” value, s.t. $\epsilon(\gamma) = \max_{\pi \in \Pi_{\gamma}} (|E_{\pi}| - |O_{\gamma}|)$.

To maximise probability of generalisation, we use the weakness proxy *and* try to maximise utility. Increasing utility changes the abstraction layer to allow construction of weaker correct policies. Beyond a certain point, utility may be increased by increasing $|L_{\mathbf{v}}|$ without actually changing the weakest correct policies Π_{γ} contains (just the size of their extensions). This means it is not *always* necessary to increase utility to construct policies that generalise, but it is necessary up to a point. Utility is maximised when $\mathbf{v} = P$, though in practice finite resources would limit us to smaller vocabularies. Γ_P contains all tasks in all vocabularies. Hence, for every task ρ in Γ_P we can define a function that takes a vocabulary \mathbf{v} and returns a \mathbf{v} -task which is a child of ρ . This lets us represent a task in different vocabularies to compare their utility.

Definition 7 (uninstantiated-tasks). The set of all tasks with no abstraction (meaning $\mathbf{v} = P$) is Γ_P (it contains every task in every vocabulary). For every P -task $\rho \in \Gamma_P$ there exists a function $\lambda_{\rho} : 2^P \rightarrow \Gamma_P$ that takes a vocabulary $\mathbf{v}' \in 2^P$ and returns a \mathbf{v}' -task $\omega \sqsubset \rho$. We call λ_{ρ} an **uninstantiated-task**. It is instantiated by choosing a vocabulary.

Proposition 1 (upper bound). *If we allow infinitely large vocabularies, the most ‘intelligent’ choice of policy and vocabulary given uninstantiated task λ_ρ is:*

$$\pi \text{ and } \mathbf{v} \text{ s.t. } \mathbf{v} \in \arg \max_{\mathbf{v} \in 2^P} \epsilon(\lambda_\rho(\mathbf{v})) \text{ and } \pi \in \Pi_{\lambda_\rho(\mathbf{v})} \text{ and } \neg \exists \pi' \in \Pi_{\lambda_\rho(\mathbf{v})} (\pi <_w \pi')$$

Proof: We have equated intelligence sample efficient generalisation. According to [7, prop. 1, 2] the weakest correct policies have the highest probability of generalising. Given an uninstantiated task λ , we can use utility ϵ to measure the weakness of the weakest correct policies in different vocabularies. By choosing \mathbf{v} s.t. $\epsilon(\lambda(\mathbf{v})) = \epsilon(\lambda(P))$ we instantiate λ in a vocabulary that maximises the weakness of correct policies for λ . Then, using weakness proxy, we can select a policy that has the highest possible probability of generalising, and thus maximise sample efficiency. ■

In practice we can only build systems with finite vocabularies. Furthermore, too large a vocabulary \mathbf{v} could make \mathbf{v} -tasks intractable. However, when the vocabulary is \mathbf{v} s.t. $\epsilon(\lambda_\rho(\mathbf{v})) = \epsilon(\lambda_\rho(P))$, there must exist $\pi \in \Pi_{\lambda_\rho(\mathbf{v})}$ which is a weakest policy in $\Pi_{\lambda_\rho(P)}$. By maximising ϵ we make it more likely that there exists a weakest policy in $\pi \in \Pi_{\lambda_\rho(\mathbf{v})}$ that is also a weakest policy in $\Pi_{\lambda_\rho(P)}$.

Concluding remarks: Here we proposed computational dualism, and presented an argument in support of an alternative which allows for objective claims regarding behaviour. We proved an objective upper bound for intelligent behaviour, but to attain this upper bound one would need to search infinitely many, infinitely large vocabularies. We anticipate these results may further our understanding of AI safety and general intelligence.

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