

Modeling of a Polarity Multiplexed QPSK Optical Coherent Transmission system with Linear Impairments, Filtering and Performance Measurement

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Abstract—As the bit rate of the optical transmission system gets higher and higher, 40 Gb/s to 100 Gb/s, the modeling of proposed modulation techniques is very important so as to avoid costly practical demonstration. A Polarity Multiplexed-QPSK transmission system is modelled in MATLAB and simulated with Additive White Gaussian Noise (AWGN) added through the channel with Chromatic Dispersion (CD). Output is filtered and performance is measured and presented in this article.

Index Terms—Coherent Optical Transmission, Polarity Multiplexing, Quadrature Phase Shift Keying (QPSK).

I. INTRODUCTION

Initial optical transmission systems with Intensity Modulated Direct Detection methodology has reached the saturation level with the limitations in electronics used. Therefore, new methods were explored to support the high bit rates and long haul transmission. There are three major options considered to improve the capacity-distance product [1]. First, by incrementing the spectral efficiency. Spectral efficiency may be increased by using larger signal constellations that transmit more bits per symbol. Polarity Multiplexed Quadrature-Amplitude Modulation (PM-QAM) with Coherent detection [2] has become the modulation format for capacity increasing problem. In this article a coherent optical system is simulated with random bit generation and adding channel Additive Gaussian noise for amplitude and phase noise. Simulation is done, in order to visualize the constellation diagrams with each type of noise with variation of noise variance with Signal to Noise (SNR) ratio ranging up to 20dB. Variation with Laser Line width is also plotted. Data recovery and filtering methods were also included in the simulation. Moving average filter, weighted coefficient filter and Weiner filter with weighted coefficients were simulated and performance curves were plotted for various combinations. The results are average of repeated simulations for over hundred times with the available computer processing power and time

II. METHODOLOGY

The fiber optic channel is modeled in MATLAB with the Transmitter generating adequate number of random bits using the function “randi” which generates uniformly distributed pseudo random integers to be transmitted over the Single Mode fiber optical channel.

TABLE I
BIPOLAR BIT ASSIGNMENT FOR RANDOM GENERATED BIT

Function Generated Bit	Assigned bipolar bit
1	1
0	-1

TABLE II
QUADRANT ASSIGNMENT FOR CONSTELLATION POINT

Bipolar coded symbol coordinates	Quadrant of the constellation point
1 , 1	0
-1 , 1	1
-1 ,-1	2
1 ,-1	3

A. Modulation and Transmitter

A special type of QAM (Quadrature Amplitude modulation) with equidistant phases and constant amplitude derives the QPSK (Quadrature Phase Shift Keying) modulation format. It uses the same carrier frequency with 90 degree phase shift. Uses 2-bits per symbol and symbol positions in constellation diagram can be defined as [2],

$$C_{QPSK} = \sqrt{2} e^{j(\frac{\pi}{2}n_t - \frac{\pi}{4})} \quad ; n_t = 1, 2, 3, 4 \quad (1)$$

The “N” number of generated bits of 1s and 0s are bipolar coded to be 1s and (-1) s. It is assigned to a matrix of [N/2, 2] making the adjacent two bits occupy the same row, starting from first bit.

Based on the two adjacent bits the quadrant of the constellation point is found and stored in an array in the same row sequence.

Then the x_i bit is differentially encoded as follows by XOR operation,

$$y_i = y_{i-1} \oplus x_i \quad (2)$$

at the receiver, it is recovered by,

$$x_i = x_{i-1} \oplus y_i \quad (3)$$

Differential coded quadrant number is being represented by a complex number (Gray coded) relevant to constellation point for QPSK Modulation.

B. Fiber Propagation in Linear domain

The low pass equivalent frequency response of the optical fiber, noted as $H(f)$ has a parabolic phase profile and can be modeled by the following equation [3] ,

$$H(f) = e^{-j\alpha f^2} \quad (4)$$

where , $\alpha = \pi D \lambda^2 \frac{L}{c}$ L -Length of the fiber , λ -Operating wave length of fiber , D- CD , c- Speed of light ,L is the length of the fiber.

The parabolic phase profile is the result of the chromatic dispersion of the optical fiber [4]. The 3rd order dispersion factor β_3 is not considered in this transfer function of the fiber due to negligible effects on 40Gb/s transmission systems. However, if the transmission bit rate is higher than 40Gb/s, the β_3 should be taken into account.

- 1) Additive White Gaussian Noise(AWGN)- For both Amplitude and Phase of the signal. Using MATLAB “awgn” function for each SNR .

$$y = \text{awgn}(x, \text{snr}, 'measured')$$

- 2) Adding Chromatic Dispersion (CD) at distance L km for D (17ps/nm.km) , Speed of EM Wave , c (2.998x10⁸m/s) for the Optical operating Wave Length λ (1550nm) .CD can be modeled as low pass filter with the transfer function [7],

$$H(f) = e^{-j\alpha f^2}$$

where , $\alpha = \pi D \lambda^2 \frac{L}{c}$

- 3) Phase noise is added to signal with AWGN and CD
Phase Noise is calculated by first finding σ from [5],

$$\sigma = \sqrt{\frac{2\pi(LW)}{R}} \quad (5)$$

where, LW-Line Width of Laser (typical - 1 Mhz to 20 MHz) ,R-Data bit rate. Where, LW is the Line Width of the Laser used (Typical 1MHz to 20Mhz) and R is the data rate of transmission. AWGN added signal is multiplied by $e^{j\sigma v}$, where $v = \text{randn}()$ from matlab.(“randn” returns normally distributed random numbers) to generate the phase noise added signal.

C. Data recovery

In data recovery, fourth power of the received signal is taken to eliminate the QPSK modulation. In data recovery, fourth power of the received signal is taken to eliminate the QPSK modulation.

$$C_{QPSK} = \sqrt{2} e^{j(\frac{\pi}{2}n_t - \frac{\pi}{4})} \quad ; n_t = 1, 2, 3, 4$$

By getting the 4th power [6],

$$\begin{aligned} C_{QPSK}^4 &= \sqrt{2}^4 \left[e^{j(\frac{\pi}{2}n_t - \frac{\pi}{4})} \right]^4 \\ &= 4 e^{j(2\pi n_t - \pi)} \\ &= -4 \quad \text{for } n_t = 1, 2, 3, 4 \end{aligned} \quad (6)$$

If the received phase angle after quadrupled manipulation of i^{th} symbol is ϕ_i ,the output angle will be $4\phi_i$. Therefore, output need to be divided by four to recover the received symbol phase angle with the negative sign.The resulting frequency-quadrupled carrier components are added and then low pass filtered for further noise suppression.

D. Filtering

Quadrupled signal is being sent through filters for further noise suppression. Filters used in this simulation are,

- Average Filter (Equal Coefficients , order 4)
- Weighted coefficient filter(order 4)
- Wiener Filter(Viterbi coefficients)
- Average filter followed by Weighted Coefficient filter(Cascaded)
- Average Filter followed by Weiner filter (Cascaded)

following is the description in detail,

- 1) *Average Filter:* Average filter of order 4 is simulated for received symbol after quadrupling operation.Average filter (FIR or non-recursive) transfer function is [6],

$$H(Z) = \frac{1}{M} \sum_{n=0}^{M-1} Z^{-n}$$

For Order 4 filter M=5,

$$H(Z) = \frac{1}{5} \sum_{n=0}^4 Z^{-n}$$

$$H(Z) = \frac{1}{5} [Z^{-4} + Z^{-3} + Z^{-2} + Z^{-1} + 1]$$

Impulse response of the filter is,

$$h[n] = \begin{cases} 1/5 & , 0 \leq n \leq 4 \\ 0 & , \text{other wise} \end{cases}$$

Its Frequency response is given by ,

$$\begin{aligned} H(e^{j\omega}) &= \frac{1}{5} \sum_{n=0}^4 H(e^{-j\omega n}) \\ &= \frac{1}{5} \frac{1 - e^{-j5\omega}}{1 - e^{-j\omega}} \\ &= \frac{1}{5} \frac{\sin(\frac{5\omega}{2})}{\sin(\frac{\omega}{2})} e^{-j(4)\omega/2} \end{aligned}$$

So the magnitude response of transfer function is,

$$|H(e^{j\omega})| = \left| \frac{1}{5} \frac{\sin(\frac{5\omega}{2})}{\sin(\frac{\omega}{2})} \right|$$

and the phase response of the transfer function is,

$$\theta(\omega) = -\frac{(4)}{2}\omega$$

$$\left(-\frac{d\theta}{d\omega}\right)_{\omega=\omega_0} = \frac{4}{2}$$

Therefore, Group delay, $\tau(\omega)$ is given by,

$$\tau(\omega) = 2$$

For the Average filter of order 4 (M=5) , Group delay is 2.

2) *Weighted Coefficient Filter*: Instead of having equal coefficients , a set of coefficients or weightages are used in this case. In this simulation More weightage is given to most recent bit. Here also order 4 filter is used to compare with the average filter case.

$$H(Z) = \sum_{n=0}^{M-1} a_n Z^{-n}$$

where, $\sum_{n=0}^{M-1} a_n = 1$
for the order 4 weighted average filter,

$$H(Z) = \sum_{n=0}^4 a_n Z^{-n}$$

coefficients can be written in the form $a_n = b_n / \sum_{n=0}^4 b_n$ and , $b_n = 2b_{n+1}$ is used to calculate the coefficients , as it is absolutely necessary to normalize them. By having the last coefficient as unity (i.e $a_4 = 1$).

then, $b_4 = 1$, $b_3 = 2, b_2 = 4, b_1 = 8, b_0 = 16$
to normalize,

$$\sum_{n=0}^4 b_n = 31$$

therefore,

$a_4 = 1/31$, $a_3 = 2/31, a_2 = 4/31, a_1 = 8/31, a_0 = 16/31$
Order 4 filter is implemented with transfer function,

$$H(Z) = \sum_{n=0}^4 a_n Z^{-n}$$

$$H(Z) = \left[\frac{16}{31} + \frac{8}{31}Z^{-1} + \frac{4}{31}Z^{-2} + \frac{2}{31}Z^{-3} + \frac{1}{31}Z^{-4} \right]$$

3) *Wiener Filter*: Wiener filter equation can be shown as follows,

$$V_i = \sum_{n=-N}^{+N} v_i \cdot U_{i-n}$$

where, N - Filter Half width ,U- Filter input, V_i = Filter i_w^{th} output

at the code level , it is implement by calculating the laser noise variances for amplitude and phase.Wiener coefficient is further optimized by introducing weighted Wiener coefficients.Wiener coefficients v_i are calculated inside the Matlab code as described in previous section.

4) *Cascaded Filter Simulation*: Simulation is done for ,

- 1) Average Filter followed by Weighted coefficient filter
- 2) Average filter followed by Weiner filter

From the Matlab code, output from the first filter is fed to the second filter to obtain the cascaded result.

Cascaded system transfer function can be derived as,

$$H(Z) = H_1(Z) * H_2(Z) \quad (7)$$

where , $H_1(Z)$ and $H_2(Z)$ are respective individual system responses.

III. RESULTS

A. Constellation diagrams

QPSK constellation diagrams with Data bit rates from 1 kb/s to 1 Gb/s , Signal to Noise Ratio (SNR) from 0 dB to 20 dB and Laser line widths from 1 MHz to 20 MHz were the changing parameters.

1) *Impact of SNR in constellation with same Phase Noise Variance*: Next shows the constellation diagram with the Phase noise variance (σ) at 0.17725 for the data transmission rate of 1 Gb/s (with laser Linewidth at 5MHz) for SNR values of 12dB,16dB and 20 dB.It is observed that, the spread of the received phase noise increases with the decreased SNR value. Received AWGN concentrated around the transmitted bit, more or less in a circular shape with mean radii inversely proportional to SNR value.

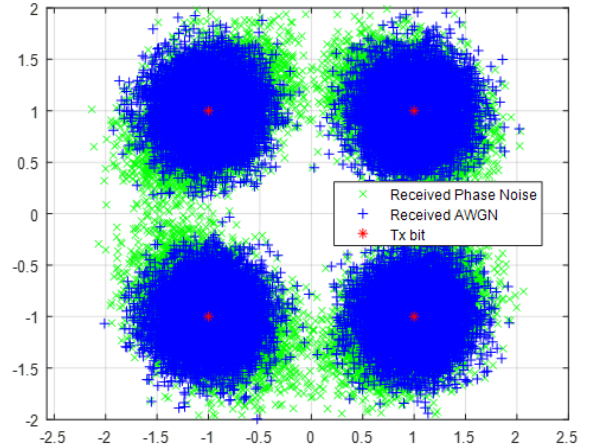


Fig. 1. QPSK constellation diagram at the receiver,LW at 5 MHz,SNR 12 dB ,R 1 Gb/s.

2) *Impact of data Transmission rate in constellation with same SNR value* : It is observed that when increasing the data transmission bit rate, recovery gets more and more difficult due to the scattering of received bits adding more error bits.

IV. EYE DIAGRAM COMPARISON WITH FIBER DISTANCE

Accordingly, with the above parameters simulation is carried out for fiber distance of 2km, 5km, 10km and 40 km from the transmitter. It is observed that with the increased distance,

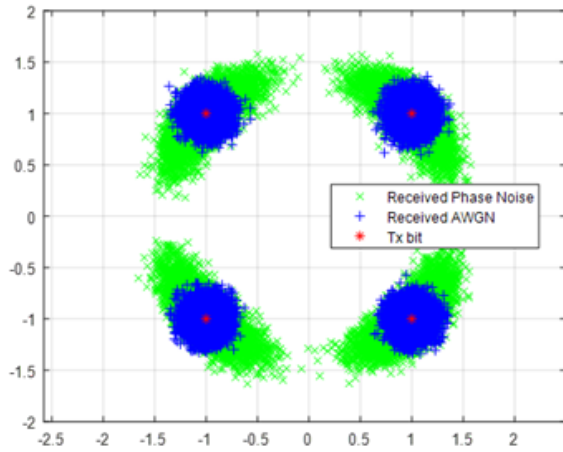


Fig. 2. QPSK constellation diagram at the receiver ,Laser Line Width 5 MHz, SNR 20dB at data bit rate 1kb/s.

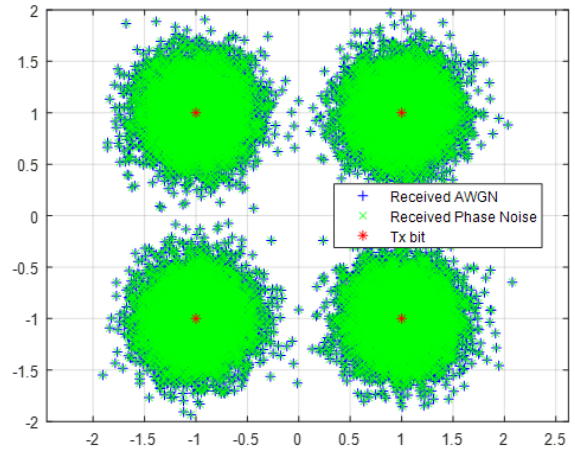


Fig. 4. QPSK constellation diagram at the receiver ,Laser Line Width 5 MHz,SNR 12 dB at data bitrate 1 Gb/s.

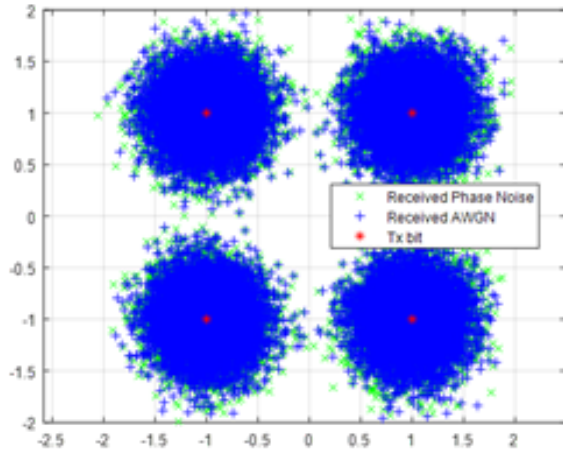


Fig. 3. QPSK constellation diagram at the receiver ,Laser Line Width 5 MHz,SNR 12 dB at data bitrate 10 Gb/s.

chromatic dispersion is increased, hence the eye is closed. However, a cyclic behavior [4] is observed in Chromatic Dispersion penalty with distance [7].

Chromatic dispersion effect variation with the data bitrate is also simulated for 5Gb/s and 10Gb/s. Eye diagrams at 5km fiber distance is shown below.

A. Bit Error Ratio (BER) Calculation

By definition Bit Error Ratio (BER) is the ration between number of error bits received to total number of bits transmitted during the selected time. In this simulation exercise , BER values are averaged over number of iterations to plot the output. This method also referred as Monte Carlo method.

1) *BER vs SNR curves with different types of filters:* Simulated results with Simple average filter, weighted coefficient filter, Weighted coefficient Weiner filter and cascaded filters with average filter with weighted setup and average filter with

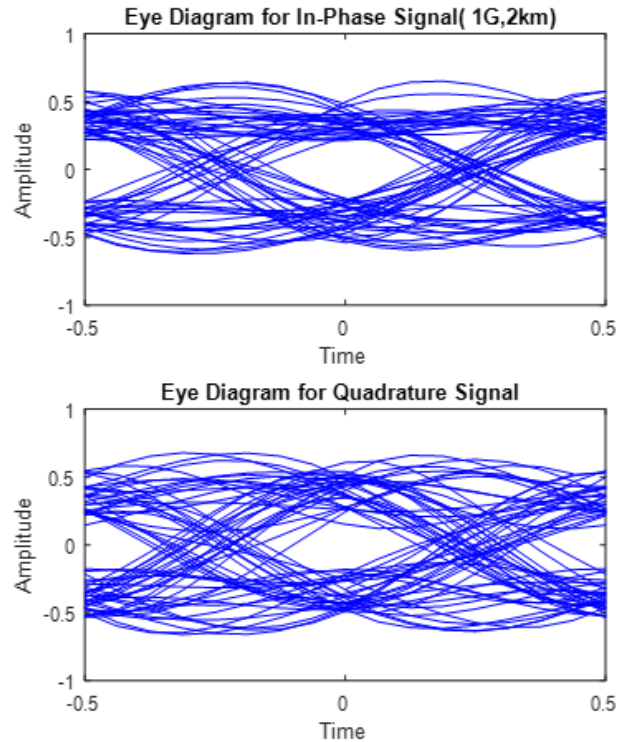


Fig. 5. Eye diagrams at fiber distance of 2km for bit rate 1 Gb/s and CD of 17 ps/nm.km.

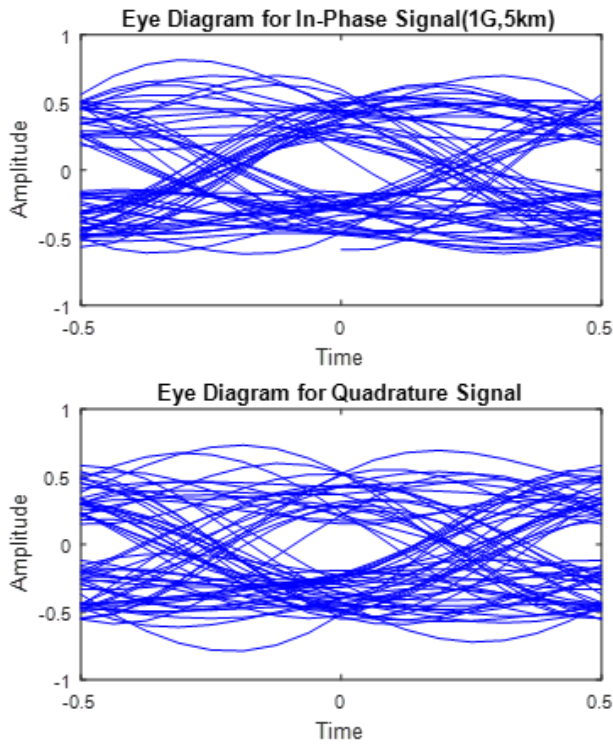


Fig. 6. Eye diagrams at fiber distance of 5km for bit rate 1 Gb/s and CD of 17 ps/nm.km

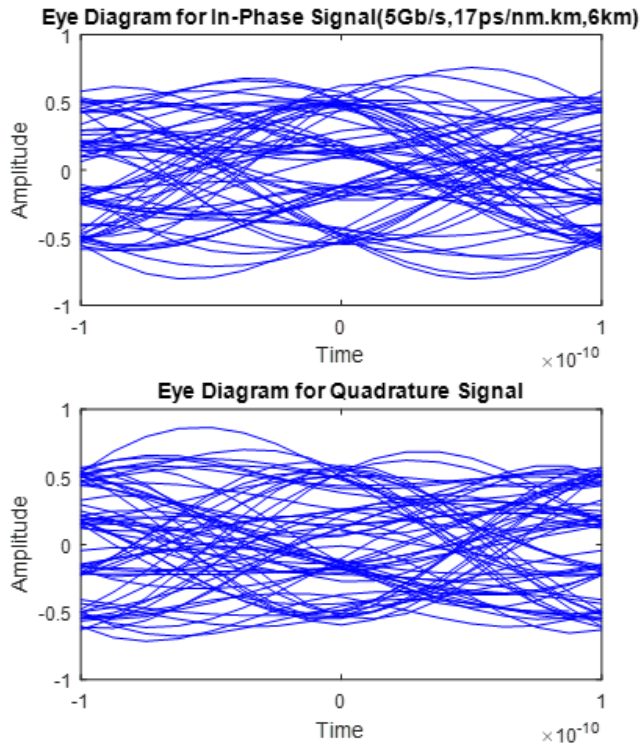


Fig. 7. Eye diagrams at 6km fiber distance for bit rates of 5 Gb/s with 17ps/nm.km CD

Weiner filter is shown below. It is observed that Weighted Wiener coefficient filter shows better BER at the SNR range 6dB to 13dB. Almost same performance available outside this region for all other filters .

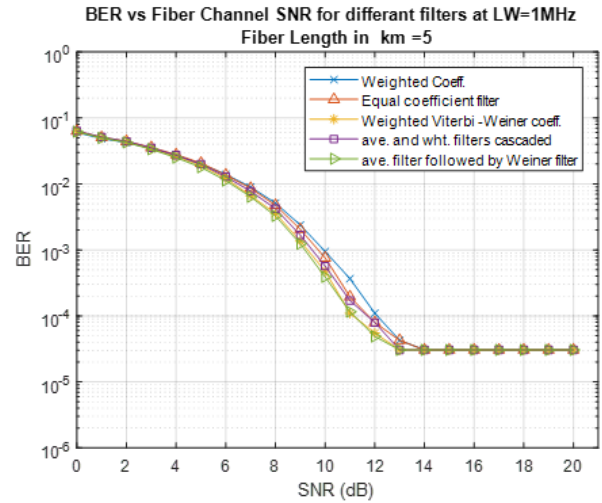


Fig. 8. BER vs Channel SNR for different filters at LW=1 MHz at fiber distance 5km and data bitrate 1 Gb/s with 17ps/nm.km CD

To analyze effect of laser line width (LW), BER vs SNR curves plotted with varying the Laser line Width in next graph. All the graphs are on top of each other since the impact is not significant with the high bitrate in Giga Hertz range. It is almost same with all the types of filters simulated, except the small variations between 8dB to 17dB SNR range , with better performances for low linewidths (1MHz). It is a limitation with commercially distributed lasers.

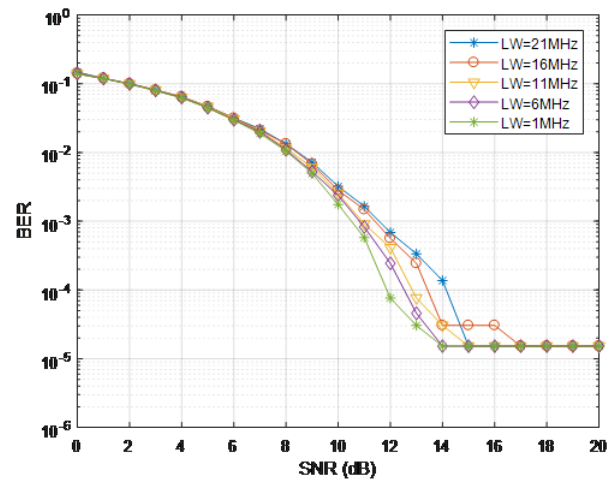


Fig. 9. BER vs Fiber Channel SNR for different Laser Line Widths with equal coefficient filter at fiber length 2km at 1 Gb/s bit rate with 17ps/nm.km CD

Fig. 4.3 shows the impact of average filter followed by Weiner coefficient filter setup at fiber length 2km at 10 Mb/s bit rate with 17ps/nm.km Chromic Dispersion.

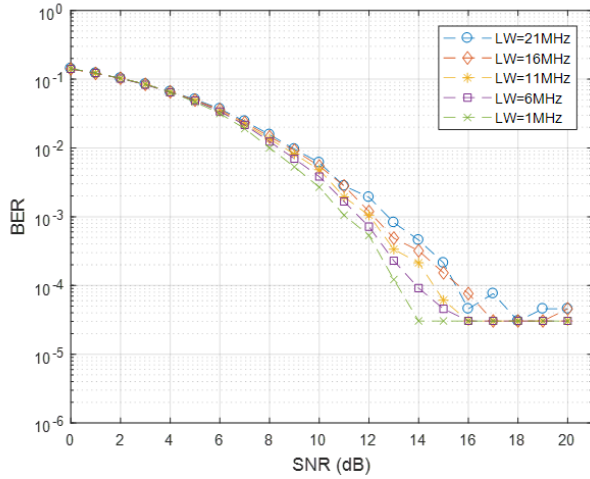


Fig. 10. BER vs Fiber Channel SNR for different Laser Line Widths with equal coefficient filter at fiber length 2km at 10 Gb/s bit rate with 17ps/nm.km CD

By changing the bitrate in to Mega Hertz range (10 Mb/s) it is visible that, with lower (narrow) Laser line widths, BER becomes to lowest possible at low SNR levels. When the Line Width of the laser increases , higher SNR levels are required to reach the minimum BER.

V. CONCLUSION

In this study comprehensive simulation study of Polarity Multiplexed QPSK system is implemented for an optical fiber transmission system. The results were presented in the form of constellation diagrams, eye diagrams and BER plots .It clearly shows the phase noise distribution extending outwards from the AWGN noise patches in a circular way centering the origin. It is observed that, the spread of the received phase noise increases with the decreased SNR value(increased noise). Received AWGN concentrated around the transmitted bit , more or less in a circular shape with mean radii inversely proportional to SNR value. Once the phase noise is high , the constellation looks like a ring , making the error count high.A cyclic behavior is observed in Chromatic Dispersion penalty with distance. When the transmission data rate is increased , relative eye closure to lower bitrate is available . Simulated results with Weighted coefficient Wiener filter shows better performance over other filters with all the linewidth combinations and transmission data rates. With the implementation of cascaded filters with average filter and Wiener filter , shows marginal improvement over Wiener filter alone. The impact of Lase Line Width is comprehensively plotted covering all the cases. With narrow Lase line widths , minimum bit error ratios are achieved with lowest SNR values.In this simulation, minimum BER reached is around 2×10^{-5} . However, once the minimum BER is reached, it remains in the same value , although the SNR is increased because of existence of phase noise.

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