ULA Fitting for MIMO Radar

Wanlu Shi, Yingsong Li, *Senior Member, IEEE,*

***Abstract*—Nowadays, sparse multiple input and multiple output (MIMO) radar design considers the difference coarray of the sum coarray (DCSC) concept and the uniform degrees of freedom is improved significantly. Current sparse MIMO arrays are designed based on transmit and receive arrays with hole-free difference coarrays (DCAs). Recently, the uniform linear array (ULA) fitting (UF) principle is proposed which aims to design sparse arrays (SAs) with desired capacity using cascaded ULAs. However, SAs designed via UF can not guarantee hole-free DCAs. In this letter, we aim to use SAs designed via UF to devise sparse MIMO arrays with low mutual coupling. Strategies of using SAs with non hole-free DCA to design sparse MIMO arrays is introduced as well. Numerical simulations verifies the good performance of the proposed sparse MIMO array in high coupling scenarios.**

***Index Terms*—Difference coarray, MIMO radar, mutual coupling, sparse array, polynomial, DOA estimation.**

I. Introduction

NOWADAYS, multiple input and multiple output (MIMO) radar draws intensive attention due to significant capacity on improving resolution, suppressing jamming and fading mitigation [1], [2]. To detect more sources than number of sensors is always an interesting problem, which relays on the uniform degrees of freedom (uDOF) greatly. In this regard, the sparse MIMO arrays is a good solution, which can use few physical sensors to generate high uDOF and hence have received increasing investigation [3]–[5].

Early sparse MIMO arrays are designed via the sum-coarray (SCA) of the transmit array and receive array [4]–[6]. Nevertheless, the SCA only shows limited improving on uDOF. With the development of nested arrays and co-primes arrays, the difference coarray (DCA) concept has been widely studied [7], [8]. By considering the difference coarray of the sum coarray (DCSC) within MIMO radar, the achievable uDOF in increased significantly [3]. Based on DCSC concept, lots sparse MIMO arrays are introduced. The sparse MIMO array proposed in [9] considers the coprime array to serve as the transmit and receive array, and realize a good DOA estimation result. However, the co-prime array based MIMO didn't achieve a high uDOF. Later, the new nested MIMO (NN MIMO) array is proposed which uses nested array as transmit and receive array, and provides a good improvement on uDOF [10]. Besides, the nested MIMO array presents a general way for sparse MIMO array design using SAs with hole-free DCA. In [11], the sparse MIMO array based on nested-coprime array with displaced subarrays (CADiS) (NC-MIMO) has been proposed. In comparison with the NN MIMO, the NC-MIMO is less sensitive to the mutual coupling. However, NC-MIMO needs the number of sensors in receive array is multiple of 3, which in fact is unpleasant in application. In [12], the generalized NC-MIMO (GNC-MIMO) is proposed which generalizes the number of sensors in receive array in NC-MIMO to multiple of any integer. Additionally, a generalized nested MIMO (GEN-MIMO) is proposed with the same idea of NN MIMO with improved uDOF [13].

One can tell that the existing sparse MIMO arrays are designed based on SAs with hole-free DCAs, and few sparse MIMO array shows good performance in high coupling scenarios. From increasing the uDOF and reducing the mutual coupling perspective, the uniform linear array (ULA) fitting (UF) principl is consisted to built the sparse MIMO array. The UF principle is proposed to design sparse arrays (SAs) with desired property using combination of sub-ULAs, and two specific SAs (UF-3BL and UF-4BL) are devised with low mutual coupling [14]. Further, the improved UF scheme is proposed to increased the available uDOF [IUF]. In this letter, the core concern is how to use UF idea to design sparse MIMO arrays with high uDOF and low mutual coupling. One problem is that though the SAs designed via the UF principle have significant capacity, they may not have hole-free DCA.

To conquer this problem, we present two strategies to design sparse MIMO arrays using SAs with holes in DCA. The first strategy follows the method in [10], which utilizes the consecutive part in array DCA to design the sparse MIMO array. The second strategy considering to pad the holes in array DCA using the sensors beyond the consecutive part, and hence, the achievable uDOF is increased significantly. The following are our contributions.

1) The strategies of designing sparse MIMO array using UF principle is discussed. Particularly, the case when the transmit array have a DCA with holes is consisted. The strategies are valid for any SAs designed via UF principle.
2) The devised sparse MIMO arrays enjoys the merit of UF principle, i.e., they will have closed-form expressions.
3) A specific sparse MIMO array configuration is proposed with increased uDOF and low mutual coupling. The closed-form expression and optimal parameter selection are given in detail.

II. Signal Model

A. The DCSC of MIMO Radar

Consider a M-sensor colocated MIMO radar, with M_t sensors in transmit array and M_r sensors in receive array. The

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Wanlu Shi is with the College of Information and Communication Engineering, Harbin Engineering University, Harbin 150001, China. Yingsong Li is with the Key Laboratory of Intelligent Computing and Signal Processing Ministry of Education, Anhui University, Hefei, Anhui, China (e-mail: liyingsong@ieee.org)*(Corresponding author: Yingsong Li)*

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transmit and receive array have the following position set

$$\mathbb{T} = \{T_i,\, i = 1, \cdots M_t\}\,, \tag{1}$$

and

$$\mathbb{R} = \{R_j,\, j = 1, \cdots M_r\}\,. \tag{2}$$

In this way, the polynomial model for the transmit and receive array can be written as [15]

$$P_T(x) = x^{T_1} + \cdots + x^{T_{M_t}}, \tag{3}$$

and

$$P_R(x) = x^{R_1} + \cdots + x^{R_{M_r}}, \tag{4}$$

Using (3) and (4), the SCA of the MIMO array can be computed as

$$P^{\text{SCA}}_{\text{MIMO}}(x) = P_T(x) \times P_R(x). \tag{5}$$

Considering the DCA of (5) one can obtain the DCSC of the MIMO array, which can be written as

$$\begin{aligned} P^{\text{DCSC}}_{\text{MIMO}}(x) &= P^{\text{SCA}}_{\text{MIMO}}(x) \times P^{\text{SCA}}_{\text{MIMO}}(x^{-1}) \\ &= P_T(x)P_R(x)P_T(x^{-1})P_R(x^{-1}) \\ &= P_T(x)P_T(x^{-1})P_R(x)P_R(x^{-1}) \\ &= P^{\text{DCA}}_{T} P^{\text{DCA}}_{R}. \end{aligned} \tag{6}$$

It can be concluded from (6) that the DCSC of a MIMO radar is in fact the sum-coarray of the two difference-coarray origined from the transmit and receive array, respectively [10], [11].

Further, the polynomial model is utilized to analyze the SCA between two ULAs. In this letter, ULAs are expressed as $\{I, S, N\}$, where I means initial position, S denotes the inter-element space, and N stands for the number of sensors. Considering two ULAs, where ULA A has parameters $\{I_A, S_A, N_A\}$ and polynomial $P^{\text{A}}_{\text{ULA}}(x)$, and ULA B possesses parameters $\{I_B, S_B, N_B\}$ and polynomial $P^{\text{B}}_{\text{ULA}}(x)$. In this regard, the SCA between ULA A and ULA B can be written as

$$\begin{aligned} &P^{\text{A}}_{\text{ULA}}(x)P^{\text{B}}_{\text{ULA}}(x) \\ &= x^{I_A+I_B}(x^0 + \cdots + x^{(N_A-1)S_A}) \\ &\quad\times (x^0 + \cdots + x^{(N_B-1)S_B}). \end{aligned} \tag{7}$$

Following (7), the SCA can be regarded as a duplicate and transfer procedure, where one ULA serves as the prototype array to be duplicated and transferred and the other ULA is the transfer array decides transfer period and times. This point is in fact quite similar to that of the DCA discussed in [14]. Reconsidering (6), in this letter, the DCA of the transmit array is selected as the prototype array, while the DCA of the receive array is the transfer array.

B. Design Strategy

Based on (7), the following proposition is concluded.

Proposition 1: If the unit element spacing of the transfer array is larger than the aperture of the prototype array, then each period of the corresponding SCA possesses the same number of sensors with the prototype array.

In this letter, the unit element spacing of the transfer array is S_{t} and the number of sensors in transfer array is N_{t}, where the unit element spacing of the prototype array is S_{p} and the number of sensors in transfer array is N_{p}. uDOF of prototype array is uDOF_{p} and the final uDOF of MIMO is $\text{uDOF}_{\text{MIMO}}$. Proposition 1 is quite straightforward if we set $S_B > (N_A - 1)S_A$ in (7). In fact, if S_{t} is selected as

$$S_{\text{t}} = \text{uDOF}_{\text{p}} + 1, \tag{8}$$

a hole-free ULA in DCSC can be realized. For instance, existing sparse MIMO arrays, such as nested MIMO, NC-MIMO, GNC-MIMO, and GEN-MIMO are both designed via (8) [10]–[13].

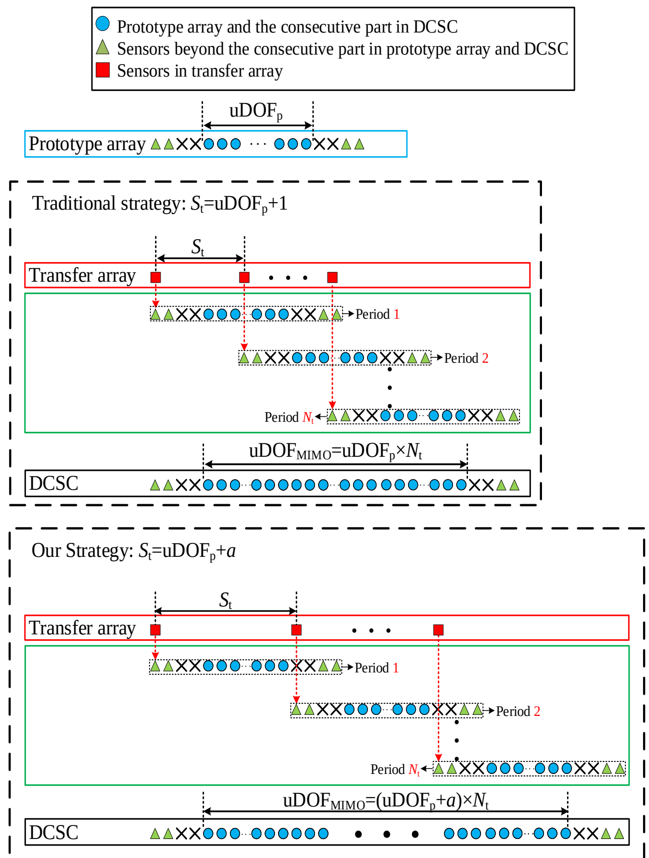


Fig. 1: Illustration of the design strategies of ULA fitting MIMO.

In this letter, the objective is to design sparse MIMO array with not only high uDOF but also low mutual coupling. In this regard, the UF principle is a good choice. In [14], the basic UF principle is discussed in detail and two SAs with low mutual coupling (UF-3BL and UF-4BL) are devised. Besides, the improved UF scheme is proposed in [IUF], which is able to design SAs with high uDOF (ATLI-1BL). Now, follow the idea of ULA fitting, we continue to design sparse MIMO array with combined ULAs, and the strategies are shown in Fig. 1.

Due to the fact that the SAs designed via UF can not guarantee a hole-free DCA, therefore, to solve this problem, two strategies are presented in Fig. 1. The first strategy is the *Traditional Strategy*, which merely uses the consecutive part

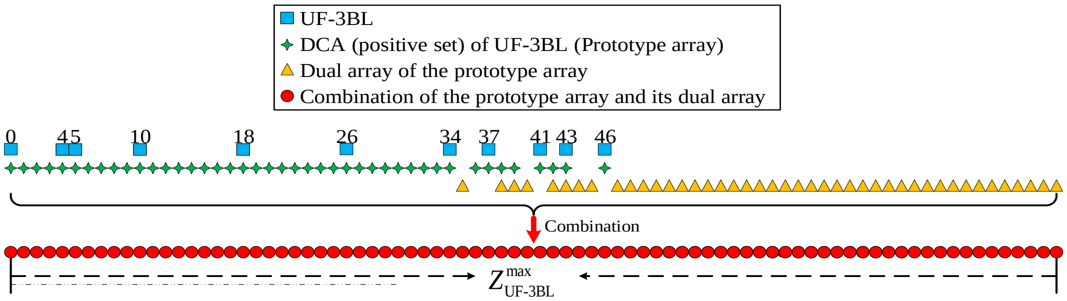


Fig. 2: Illustration of the basic MIMO array design via UF-3BL utilizing *Our Strategy*.

in DCA and the other sensors are abandoned. *Our Strategy* aims maximizing the unit element spacing in transfer array, and uses sensors beyond the consecutive part in central ULA of the adjacent period (green triangle in Fig. 1) to pad the holes in current period. By using *Our Strategy*, the unit element spacing of the transfer array is set as $S_{\rm t} = {\rm uDOF_p} + a$, where a is a offset value. In doing this, two benefits can be obtained. First, we can fully take advantage of the low mutual coupling character of UF based SAs. Second, *Our Strategy* can enlarge the unit element space than that in *Traditional Strategy*, so that the final uDOF is improved significantly. Based on the fact that the DCA is a symmetric structure, *Our Strategy* can be formulated as the following problem

$$\begin{aligned} &\arg\max_{\alpha}\ Z \\ &\text{s.t. cons.}\{P_{\text{DCA}}(x) + \alpha P^{\text{dual}}_{\text{DCA}}(x)\} = P_{\text{cons.}}\{Z\}, \end{aligned} \tag{9}$$

where $P_{\text{DCA}}(x)$ indicates the DCA (positive set) of an SA, $P^{\text{dual}}_{\text{DCA}}(x)$ implies the dual array (the detailed definition of dual array can be found in [14]) of $P_{\text{DCA}}(x)$, cons.$\{\cdot\}$ returns the consecutive part of a polynomial and omits the coefficients (for example, cons.$\{x^1 + 3x^2 + 4x^5\} = x^1 + x^2$), and $P_{\text{cons.}}\{Z\}$ has the following expression

$$P_{\text{cons.}}\{Z\} = x^0 + x^1 + \cdots + x^Z. \tag{10}$$

III. Design of UF MIMO

In this section, an example of designing sparse MIMO array using UF principle is presented. The UF-3BL is selected as transmit array and the ATLI-1BL is chosen as the receive array, the devised sparse MIMO array is termed as UF3-ATLI1BL MIMO.

A. Design of UF 3BASE MIMO

The UF–3BL is proposed in [14], of which the mutual coupling between adjacent sensors is reduced greatly. The closed-form expression for UF-3BL can be summarized as

$$\left\{\begin{array}{l} \text{P-1}: \{0, 3, M_{\rm b}\}, \\ \text{P-2}: \{3M_{\rm b}+1, 1, 2\}, \\ \text{P-3}: \{6M_{\rm b}+4, 3M_{\rm b}+5, M_{\rm t}\}, \\ \text{P-4}: \{3M_{\rm t}M_{\rm b}+5M_{\rm t}+3M_{\rm b}+2, 3, M_{\rm b}\}, \\ \text{P-5}: \{3M_{\rm t}M_{\rm b}+5M_{\rm t}+6M_{\rm b}+3, 2, 2\}, \\ \text{P-6}: \{3M_{\rm t}M_{\rm b}+5M_{\rm t}+6M_{\rm b}+8, 3, M_{\rm b}\}, \end{array}\right. \tag{11}$$

with

$$\left\{\begin{array}{l} M_{\rm b} = \lfloor \frac{M_t-5}{6} \rfloor, M_t \geq 11, \\ M_{\rm t} = M_t - 3N_{\rm b} - 4, \end{array}\right. \tag{12}$$

where M_t is the number of sensors in transmit array and $\text{P}-i$ are ULAs. In Fig. 2, the method of using UF-3BL to design sparse MIMO array based on *Strategy 2* is presented, where $S_{\rm t} = Z^{\max}_{\text{UF-3BL}}$ is the unit element spacing for the receive array. Based on (9), to obtain the feasible value of $Z^{\max}_{\text{UF-3BL}}$ one should first find the exact expression for the DCA of UF-3BL. Based on [14], the DCA of UF-3BL have the following expressions, when $M_{\rm b} = 1$,

$$\begin{aligned} P^{\text{DCA}}_{\text{UF-3BL}}(x) = &\underbrace{x^0 + \cdots + x^{8M_{\rm t}+2}}_{\text{consecutive}} + \underbrace{x^{8M_{\rm t}+4} + \cdots + x^{8M_{\rm t}+7}}_{\text{consecutive}} \\ &+ \underbrace{x^{8M_{\rm t}+9} + \cdots + x^{8M_{\rm t}+11}}_{\text{consecutive}} + x^{8M_{\rm t}+14}, \end{aligned} \tag{13}$$

and, when $M_{\rm b} \geq 2$,

$$\begin{aligned} P^{\text{DCA}}_{\text{UF-3BL}}(x) = &\underbrace{x^0 + \cdots + x^{3M_{\rm b}M_{\rm t}+5M_{\rm t}+3M_{\rm b}-1}}_{\text{consecutive}} \\ &+ \underbrace{x^{3M_{\rm b}M_{\rm t}+5M_{\rm t}+3M_{\rm b}+1} + \cdots + x^{3M_{\rm b}M_{\rm t}+5M_{\rm t}+6M_{\rm b}+5}}_{\text{consecutive}} \\ &+ x^{3M_{\rm b}M_{\rm t}+5M_{\rm t}+6M_{\rm b}+8}(x^0 + x^3 + \cdots + x^{3(M_{\rm b}-1)}). \end{aligned} \tag{14}$$

Based on [14], the dual array of $P^{\text{DCA}}_{\text{UF-3BL}}(x)$ can be given as, when $M_{\rm b} = 1$

$$\begin{aligned} P^{\text{DCA-dual}}_{\text{UF-3BL}}(x) = x^0 + &\underbrace{x^3 + \cdots + x^5}_{\text{consecutive}} + \underbrace{x^7 + \cdots + x^{10}}_{\text{consecutive}} \\ &+ \underbrace{x^{12} + \cdots + x^{8M_{\rm t}+14}}_{\text{consecutive}}, \end{aligned} \tag{15}$$

and when $M_{\rm b} \geq 2$

$$\begin{aligned} P^{\text{DCA-dual}}_{\text{UF-3BL}}(x) = &(x^0 + x^3 + \cdots + x^{3(M_{\rm b}-1)}) \\ &+ \underbrace{x^{3M_{\rm b}} + \cdots + x^{6M_{\rm b}+4}}_{\text{consecutive}} \\ &+ \underbrace{x^{6M_{\rm b}+6} + \cdots + x^{3M_{\rm b}M_{\rm t}+5M_{\rm t}+9M_{\rm b}+5}}_{\text{consecutive}}. \end{aligned} \tag{16}$$

Then, based on (13), (14), (15) and (16), the solution of (9) can be expressed as

$$\alpha = \left\{\begin{array}{ll} x^{8M_{\rm t}+3}, & M_{\rm b} = 1 \\ x^{3M_{\rm b}M_{\rm t}+5M_{\rm t}+3M_{\rm b}}, & M_{\rm b} \geq 2 \end{array}\right. \tag{17}$$

with

$$Z^{\max}_{\text{UF-3BL}} = \left\{\begin{array}{ll} 16M_{\rm t} + 17, & M_{\rm b} = 1 \\ 6M_{\rm b}M_{\rm t} + 10M_{\rm t} + 12M_{\rm b} + 5, & M_{\rm b} \geq 2 \end{array}\right. \tag{18}$$

Hence, the unit element spacing for the receive array is $Z^{\max}_{\text{UF-3BL}}$. Based on [IUF], the closed form expression for ATLI-1BL is given by

$$\begin{cases} \text{P-1}: \{0, 1, M_{\mathrm{a}}\}, \\ \text{P-2}: \{2M_{\mathrm{a}}-1, 2M_{\mathrm{a}}-1, M_{\mathrm{a}}-1\}, \\ \text{P-3}: \left\{2M_{\mathrm{a}}^2 - M_{\mathrm{a}}, 4M_{\mathrm{a}}-1, M_{\mathrm{t}}\right\}, \\ \text{P-4}: \left\{4M_{\mathrm{a}}M_{\mathrm{f}} + 2M_{\mathrm{a}}^2 - M_{\mathrm{f}} - 3M_{\mathrm{a}} + 1, 2M_{\mathrm{a}}, M_{\mathrm{a}}-1\right\}, \\ \text{P-5}: \left\{4M_{\mathrm{a}}M_{\mathrm{f}} + 4M_{\mathrm{a}}^2 - M_{\mathrm{f}} - 5M_{\mathrm{a}} + 1, 1, M_{\mathrm{a}}\right\}, \end{cases} \tag{19}$$

with

$$\begin{cases} M_{\mathrm{a}} = \left\lfloor \frac{M_{\mathrm{r}}+4}{6} \right\rfloor, \\ M_{\mathrm{f}} = M_{\mathrm{r}} - 4M_{\mathrm{a}} + 2, \end{cases} \quad M_{\mathrm{r}} \geq 14, \tag{20}$$

where M_{r} is the number of sensors in receive array.

B. uDOF of UF MIMO and Optimal Parameter Selection

The uDOF of UF3-ATLI1BL MIMO depends on the uDOF of the UF-3BL and the ATLI-1BL. Based on [14], the uDOF of UF-3BL can be expressed as, when $M_{\mathrm{t}} \geq 17$

$$\text{uDOF}_{\text{UF-3BL}} = \begin{cases} \frac{M_{\mathrm{t}}^2}{2} + 2M_{\mathrm{t}} - 11 & M_{\mathrm{t}}\%6 = 0, 4, \\ \frac{M_{\mathrm{t}}^2}{2} + 2M_{\mathrm{t}} - 9.5 & M_{\mathrm{t}}\%6 = 1, 3, \\ \frac{M_{\mathrm{t}}^2}{2} + 2M_{\mathrm{t}} - 9 & M_{\mathrm{t}}\%6 = 2, \\ \frac{M_{\mathrm{t}}^2}{2} + 2M_{\mathrm{t}} - 13.5 & M_{\mathrm{t}}\%6 = 5, \end{cases} \tag{21}$$

and when $17 > M_{\mathrm{t}} \geq 11$

$$\text{uDOF}_{\text{UF-3BL}} = 16M_{\mathrm{t}} - 107. \tag{22}$$

Based on [IUF], the uDOF of ATLI-1BL can be expressed as, when $M_{\mathrm{r}} \geq 14$

$$\text{uDOF}_{\text{ATLI-1BL}} = \begin{cases} \frac{2M_{\mathrm{r}}^2+2M_{\mathrm{r}}}{3} - 3 & M_{\mathrm{r}}\%6 = 0, 2, \\ \frac{2M_{\mathrm{r}}^2+2M_{\mathrm{r}}}{3} - 1 & M_{\mathrm{r}}\%6 = 1, 5, \\ \frac{2M_{\mathrm{r}}^2+2M_{\mathrm{r}}-1}{3} & M_{\mathrm{r}}\%6 = 2, 4, \\ \frac{2M_{\mathrm{r}}^2+2M_{\mathrm{r}}-4}{3} - 5 & M_{\mathrm{r}}\%6 = 3, \end{cases} \tag{23}$$

and when $14 > M_{\mathrm{t}} \geq 7$

$$\text{uDOF}_{\text{ATLI-1BL}} = 14M_{\mathrm{r}} - 67. \tag{24}$$

Based on (21), (22), (23) and (24), the optimal parameter selection for UF3-ATLI1BL MIMO can be written as when $22 > M \geq 18$

$$M_{\mathrm{t}} = 11, \tag{25}$$

and when $33 > M \geq 22$

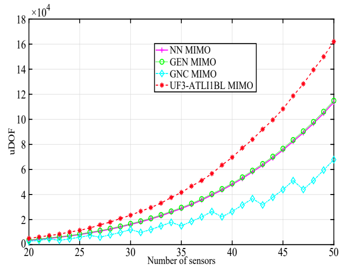
$$M_{\mathrm{t}} = \begin{cases} 12 & M = 22, 23, 26, \\ 13 & M = 24, 26, 27, 28, \\ 14 & M = 29, 30, \\ 15 & M = 31, 32, \end{cases} \tag{26}$$

and when $105 > M \geq 33$

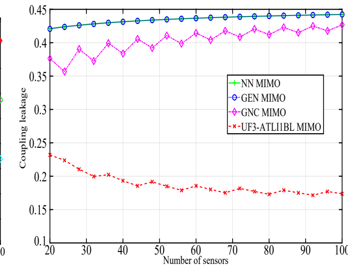
$$M_{\mathrm{t}} = \begin{cases} \left\lfloor \frac{M}{2} \right\rfloor & M\%12 = 0, 1, 9, 10, 11, \\ \left\lfloor \frac{M}{2} \right\rfloor - 1 & M\%12 = 2, 3, 4, 5, 6, 7, 8, \end{cases} \tag{27}$$

where $\lfloor \cdot \rfloor$ indicates floor operation, and when $129 > M \geq 105$

$$M_{\mathrm{t}} = \begin{cases} \left\lfloor \frac{M}{2} \right\rfloor & M\%12 = 0, 10, \\ \left\lfloor \frac{M}{2} \right\rfloor - 1 & M\%12 = 1, 2, 3, 4, 5, 6, 7, 8, \\ \left\lfloor \frac{M}{2} \right\rfloor + 1 & M\%12 = 9, 11, \end{cases} \tag{28}$$



(a) uDOF versus number of sensors.



(b) Coupling leakage versus number of sensors.

Fig. 3: uDOF and coupling leakage versus number of sensors.

and when $M \geq 129$

$$M_{\mathrm{t}} = \begin{cases} \left\lfloor \frac{M}{2} \right\rfloor & M\%12 = 0, 1, 10, \\ \left\lfloor \frac{M}{2} \right\rfloor - 1 & M\%12 = 2, 3, 4, 5, 6, 7, 8, 9, \\ \left\lfloor \frac{M}{2} \right\rfloor + 1 & M\%12 = 11. \end{cases} \tag{29}$$

The final uDOF for UF3-ATLI1BL MIMO can be given as

$$\text{uDOF} = Z^{\max}_{\text{UF-3BL}}(\text{uDOF}_{\text{ATLI-1BL}} - 1) + \text{uDOF}_{\text{UF-3BL}} - 2, \tag{30}$$

where parameters can be found in aforementioned equations.

IV. Simulations

In this section we present the simulation results of the relevant structures. Sparse MIMO array considered here are NN MIMO [10], GNC MIMO with 3 sub-ULAs in receive array [12], GEN MIMO [13] and the proposed UF3-ATLI1BL MIMO. The number of radar pulses used is 500. For all the SA geometries tested, DOAs are computed based on spatial smoothing MUSIC algorithm [7], [16].

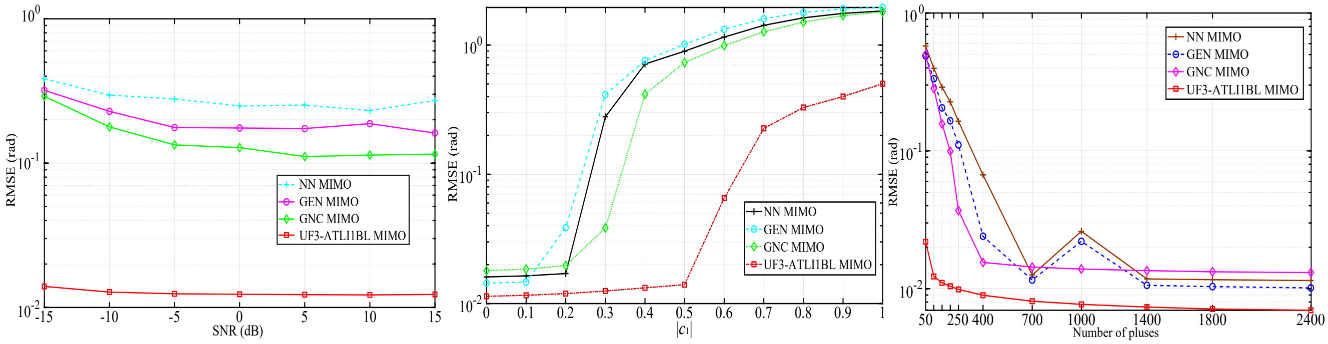
A. uDOF and Coupling Leakage

*Simulation example 1:*The uDOF and coupling leakage (see (12) in [17]) versus number of sensors is presented in Fig. 3. With the increasing number of sensors, the coupling leakage of NN MIMO, GNC MIMO and GEN MIMO increases, which indicates the mutual coupling among sensors is increased. On the contrary, the proposed UF3-ATLI1BL MIMO has much lower coupling leakage than other sparse MIMO arrays tested, which means the mutual coupling is reduced significantly. Besides, the uDOF of the proposed UF3-ATLI1BL MIMO is much higher than the relevant configurations. In the following simulations, we considering the performance of relevant structures with the influence of mutual coupling.

B. MIMO Array Performance Considering Coupling

*Simulation example 2:*The RMSE performance of the four MIMO structures are presented in Fig. 4, where all structures have 20 sensors. For NN MIMO and GEN MIMO, $M_{\mathrm{t}} = M_{\mathrm{r}} = 10$ and for the proposed UF3-ATLI1BL MIMO, $M_{\mathrm{t}} = 11, M_{\mathrm{r}} = 9$, while for the GNC MIMO, $M_{\mathrm{t}} = 5, M_{\mathrm{r}} = 15$.

The RMSE versus SNR is presented in Fig. 4 (a), where we observe 50 uniformly located targets in $[-60°, 60°]$ and $c_1 = 0.3e^{j\pi/3}$, $c_i = c_1 e^{-j(i-1)\pi/8}/i$, $i = 2, \ldots, 100$. In this



(a) RMSE performance versus SNR. (b) RMSE Performance versus $|c_1|$ (see (9) in [17]). (c) RMSE performance versus number of pulses.

Fig. 4: RMSE performance for configurations tested in different conditions.

example, one can see that the proposed UF3-ATLI1BL MIMO has a significantly improvement for all SNR values, while NN MIMO, GEN MIMO and GNC MIMO facing performance degrade. However, due to the fact that GNC MIMO stems from Generalized coprime array, it suffers less mutual coupling but still doesn't perform well.

It is also necessary to test the performance of relevant structures in various mutual coupling scenarios. We keep SNR=0 dB and vary the coupling parameter from $|c_1| = 0$ to $|c_1| = 1$, where the sources are same with the last example. It can be found from Fig. 4 (b) that the proposed UF3-ATLI1BL MIMO can provide excellent performance in both low and high coupling environment. Especially in high coupling environment, UF3-ATLI1BL MIMO shows a good improvement comparing with the existing sparse MIMO arrays.

In the last example, the RMSE performance versus number of radar pulses in high coupling environment is investigated. In this case, 30 targets are uniformly located at $-70°$ to $50°$ and the coupling parameters are set as $c_1 = 0.3e^{j\pi/3}$, $c_i = c_1 e^{-j(i-1)\pi/8}/i$, $i = 2, \ldots, 100$. From the result shown in Fig. 4 (c), it can be concluded that the UF3-ATLI1BL MIMO have the best performance. It is also worthy to note that in low radar pulses, UF3-ATLI1BL MIMO has a considerable advantage in comparison with other existing sparse MIMO arrays.

V. Conclusion

In this letter, the strategy for designing sparse MIMO array using UF principle is discussed. The proposed strategy is suitable for SAs designed via UF principle, especially the SAs with non hole-free DCAs. The proposed strategy can make full use of all the sensors in DCSC domain and enlarge the final uDOF. Based on the proposed strategy, the sparse MIMO array design problem is equivalent to solve a polynomial equation. The UF-3BL and the ATLI-1BL are selected as the transmit and receive arrays to design a sparse MIMO array with not only low mutual coupling but also high uDOF, and finally the UF3-ATLI1BL MIMO is devised. Simulation results verify the superiority of the proposed UF3-ATLI1BL MIMO, which attest the effectiveness of the proposed strategy.

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