ULA Fitting for MIMO Radar

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***Abstract*—Uniform linear array (ULA) fitting (UF) principle is a newly proposed sparse array (SA) design scheme that aims to design SAs using cascaded ULAs, based on which a number of SAs with desired capacity are devised. In this letter, we aim to introduce the UF idea to devise sparse MIMO arrays with reduced mutual coupling and increased uniform degrees of freedom. However, existing sparse MIMO array design approach needs the transmit and receive arrays in MIMO radar have hole-free difference coarrays (DCAs), which is no longer the best choice for UF principle due to the fact that SAs designed via UF cannot guarantee hole-free DCAs. In this regard, considering the difference coarray of the sum coarray (DCSC) concept, a novel sparse MIMO array design strategy which is suitable for UF principle is proposed. Based on the novel strategy and UF principle, a new sparse MIMO array is designed. Numerical simulations verify the good performance of the proposed sparse MIMO array in high coupling scenarios.**

***Index Terms*—Difference coarray, MIMO radar, mutual coupling, sparse array, polynomial, DOA estimation.**

I. INTRODUCTION

MULTIPLE input and multiple output (MIMO) radar has drawn extensive attention due to its capabilities to improve resolution, suppress jamming, and alleviating fading [1], [2]. In MIMO radars, the ability to detect multiple targets depends on the uniform degrees of freedom (uDOF). Improving uDOF for MIMO radars has been an interesting research problem. In this regard, sparse MIMO array is a good candidate for achieving high uDOF while using as few transmit and receive elements as possible [3]–[5].

Existing sparse MIMO arrays are mostly designed using the sum-coarray (SCA) of the transmit array and receive array [4]–[6]. The SCA method shows only limited improvement on uDOF. With the development of nested arrays and co-primes arrays, the difference coarray (DCA) concept has been widely studied [7], [8]. By considering the difference coarray of the sum coarray (DCSC), the achievable uDOF is increased significantly [3]. Based on the DCSC concept, many sparse MIMO array designs have been introduced. The sparse MIMO array proposed in [9] uses co-prime arrays to realize a good DOA estimation performance. However, co-prime array based MIMO radar cannot achieve a high uDOF. The new nested MIMO (NN-MIMO) array presents a general way for sparse MIMO array design using sparse arrays (SAs) with hole-free DCA and achieves good improvement on uDOF [10]. In [11], based on the coprime array with displaced subarrays (CADiS), the nested-CADiS MIMO array (NC-MIMO) was proposed. In comparison with the NN-MIMO, the NC-MIMO is less sensitive to the mutual coupling between antenna elements. However, the NC-MIMO requires that the number of elements in the receive array be a multiple of 3, which may be inconvenient in practical applications. In [12], the generalized NC-MIMO (GNC-MIMO) is proposed to remove this limitation in NC-MIMO arrays. In [13], a generalized nested MIMO (GEN-MIMO) is proposed to further improved the uDOF.

Existing sparse MIMO arrays are designed using hole-free DCAs, and few sparse MIMO array designs show good performance with high antenna coupling. To reduce the mutual coupling, the uniform linear array fitting (UF) principle has been considered for sparse MIMO array designs. The UF principle is proposed to design sparse arrays (SAs) with desired property using combination of sub-ULAs, and two specific SAs (UF-3BL and UF-4BL) are devised with low mutual coupling [14]. Further, the improved UF scheme is proposed to increased the available uDOF [15].

In this letter, we aim to extend the UF principle to designing MIMO SAs with high uDOF and low mutual coupling. Different from the strategy in [10], which uses the consecutive part in array DCA for MIMO SA design, we propose a new design method by padding the holes in array DCA using the sensors beyond the consecutive part to achieve a significantly higher uDOF. The contributions of this work are:

1) The strategy of designing sparse MIMO array using UF principle is discussed. Particularly, the case when the transmit array have a DCA with holes is considered. The strategy is valid for any SAs designed via UF principle.
2) The devised sparse MIMO arrays enjoys the merit of UF principle, i.e., they will have closed-form expressions.
3) A specific sparse MIMO array configuration is proposed with increased uDOF and low mutual coupling. The closed-form expression and optimal parameter selection are given in detail.

II. SIGNAL MODEL

A. The DCSC of MIMO Radar

Consider a M-sensor co-located MIMO radar with M_t transmitters and M_r receivers. The transmit and receive arrays have the following position set

$$\mathbb{T} = \{T_i,\, i = 1, \cdots M_t\}\,, \tag{1}$$

and

$$\mathbb{R} = \{R_j,\, j = 1, \cdots M_r\}\,. \tag{2}$$

The polynomial model for the transmit and receive array can be written as [16]

$$P_T(x) = x^{T_1} + \cdots + x^{T_{M_t}}\,, \tag{3}$$

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and

$$P_R(x) = x^{R_1} + \cdots + x^{R_{M_r}}. \tag{4}$$

Using (3) and (4), the SCA of the MIMO array can be computed as

$$P^{\mathrm{SCA}}_{\mathrm{MIMO}}(x) = P_T(x) \times P_R(x). \tag{5}$$

Considering the DCA of (5), one can obtain the DCSC of the MIMO array, which can be written as

$$\begin{aligned} P^{\mathrm{DCSC}}_{\mathrm{MIMO}}(x) &= P^{\mathrm{SCA}}_{\mathrm{MIMO}}(x) \times P^{\mathrm{SCA}}_{\mathrm{MIMO}}(x^{-1}) \\ &= P_T(x)P_R(x)P_T(x^{-1})P_R(x^{-1}) \\ &= P_T(x)P_T(x^{-1})P_R(x)P_R(x^{-1}) \\ &= P^{\mathrm{DCA}}_T P^{\mathrm{DCA}}_R. \end{aligned} \tag{6}$$

It can be concluded from (6) that the DCSC of a MIMO radar is in fact the sum-coarray of the two difference-coarray originated from the transmit and receive array, respectively [10], [11].

The polynomial model can be utilized to analyze the SCA between two ULAs. In this letter, ULAs are expressed as $\{I, S, N\}$, where I denotes the initial position, S denotes the inter-element spacing, and N stands for the number of sensors. Considering two ULAs ULA A has parameters $\{I_A, S_A, N_A\}$ and polynomial $P^{\mathrm{A}}_{\mathrm{ULA}}(x)$, and ULA B possesses parameters $\{I_B, S_B, N_B\}$ and polynomial $P^{\mathrm{B}}_{\mathrm{ULA}}(x)$. In this regard, the SCA between ULA A and ULA B can be written as

$$\begin{aligned} & P^{\mathrm{A}}_{\mathrm{ULA}}(x)P^{\mathrm{B}}_{\mathrm{ULA}}(x) \\ &= x^{I_A+I_B}(x^0 + \cdots + x^{(N_A-1)S_A}) \\ &\quad \times (x^0 + \cdots + x^{(N_B-1)S_B}). \end{aligned} \tag{7}$$

Following (7), the SCA can be regarded as a duplicate-and-transfer procedure, where one ULA serves as the prototype array to be duplicated and transferred and the other ULA decides transfer period and times. This point is in fact quite similar to that of the DCA discussed in [14]. In this letter, the DCA of the transmit array is selected as the prototype array, while the DCA of the receive array serves as the duplicate-and-transfer (DT) array.

B. Design Strategy

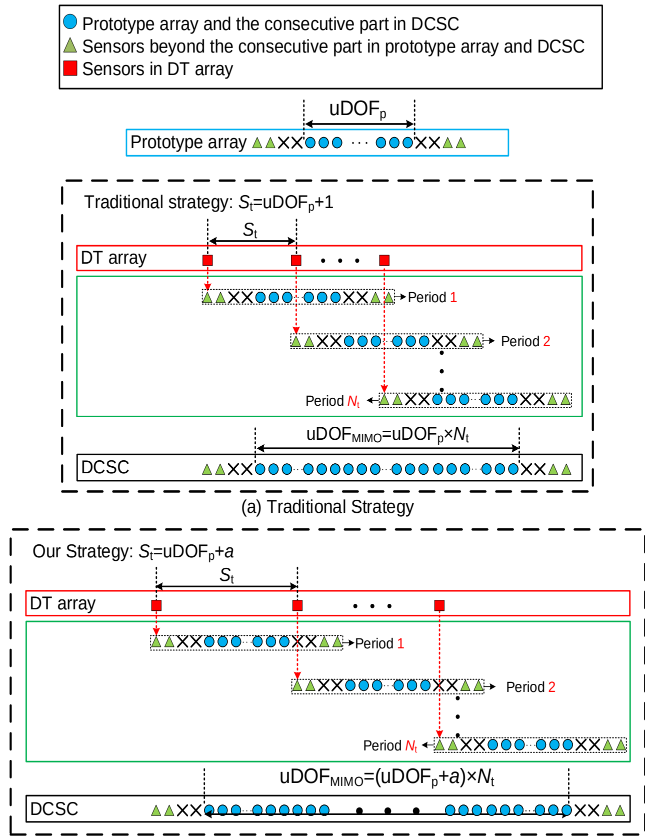
Based on (7), the following proposition is made.

Proposition 1: If the unit element spacing of the DT array is larger than the aperture of the prototype array, then each period of the corresponding SCA possesses the same number of sensors with the prototype array.

In this letter, the unit element spacing of the DT array is S_{t} and the number of sensors in DT array is N_{t}, where the unit element spacing of the prototype array is S_{p} and the number of sensors in the prototype array is N_{p}. The uDOF of prototype array is $\mathrm{uDOF_p}$ and the final uDOF of MIMO is $\mathrm{uDOF_{MIMO}}$. Proposition 1 is quite straightforward if we set $S_B > (N_A - 1)S_A$ in (7). In fact, if S_{t} is selected as

$$S_{\mathrm{t}} = \mathrm{uDOF_p} + 1, \tag{8}$$

a hole-free ULA in DCSC can be realized. For instance, existing sparse MIMO arrays, such as nested MIMO, NC-MIMO, GNC-MIMO, and GEN-MIMO are all designed via (8) [10]–[13].



(a) Traditional Strategy

(b) Our Strategy

Fig. 1: Illustration of the design strategies for sparse MIMO array.

In this letter, the objective is to design sparse MIMO array with not only high uDOF but also low mutual coupling. In this regard, the UF principle is a good choice. In [14], the basic UF principle is discussed in detail and two SAs with low mutual coupling (UF-3BL and UF-4BL) are devised. An improved UF scheme is proposed in [15] for SA designs with high uDOF (ATLI-1BL). Following the idea of UF, this paper presents further improved design methods for sparse MIMO array with combined ULAs.

SAs designed using UF cannot guarantee a hole-free DCA. To solve this problem, two strategies are possible as shown in Fig. 1. The first strategy is the *traditional strategy*, which merely uses the consecutive part in DCA and the other sensors are abandoned [10]. In contrast, *Our strategy* aims at maximizing the unit element spacing in the transfer array, and uses sensors beyond the consecutive part in the central ULA of the adjacent period (green triangle in Fig. 1) to pad the holes in current period. As such, the unit element spacing of the transfer array is set as $S_{\mathrm{t}} = \mathrm{uDOF_p} + a$, where a is an offset value. By doing this, two benefits can be obtained. First, we can fully take advantage of the low mutual coupling characteristics of UF based SAs. Second, we can enlarge the unit element space than that in [10], so that the final uDOF can be improved significantly. Based on the fact that the DCA

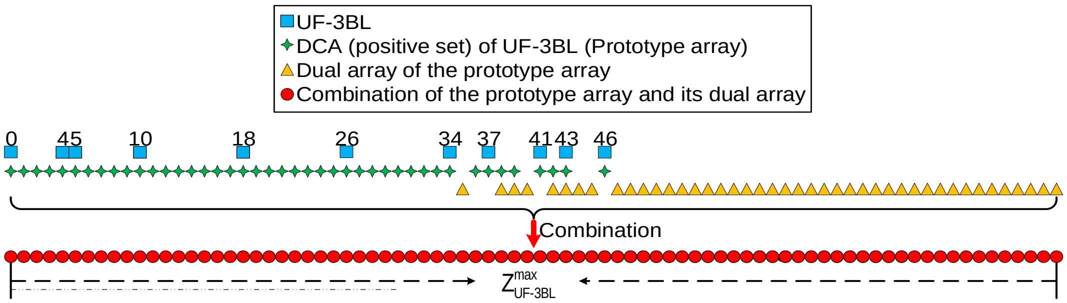


Fig. 2: Illustration of the basic MIMO array design via UF-3BL utilizing *Our Strategy*.

is a symmetric structure, *our strategy* can be formulated as the following problem

$$\begin{aligned} &\arg_{\alpha} \quad \max \ Z \\ &\text{s.t.} \ \ \text{cons.}\{P_{\text{DCA}}(x) + \alpha P^{\text{dual}}_{\text{DCA}}(x)\} = P_{\text{cons.}}\{Z\}, \end{aligned} \tag{9}$$

where $P_{\text{DCA}}(x)$ indicates the DCA (positive set) of an SA, $P^{\text{dual}}_{\text{DCA}}(x)$ implies the dual array (the detailed definition of dual array can be found in [14]) of $P_{\text{DCA}}(x)$, cons.$\{\cdot\}$ returns the consecutive part of a polynomial and omits the coefficients (for example, cons.$\{x^1+3x^2+4x^5\} = x^1+x^2$), and $P_{\text{cons.}}\{Z\}$ has the following expression

$$P_{\text{cons.}}\{Z\} = x^0 + x^1 + \cdots + x^Z. \tag{10}$$

III. Design of UF-MIMO

In this section, an example of designing sparse MIMO array using the UF principle is presented. The UF-3BL is selected as transmit array and the ATLI-1BL is chosen as the receive array, the devised sparse MIMO array is termed as UF3-ATLI1BL MIMO.

A. *Design of UF 3BASE MIMO*

The UF–3BL has been proposed in [14], in which the mutual coupling between adjacent sensors is greatly reduced. The closed-form expression for UF-3BL can be summarized as

$$\left\{ \begin{array}{l} \text{P-1}: \{0, 3, M_{\text{b}}\}, \\ \text{P-2}: \{3M_{\text{b}}+1, 1, 2\}, \\ \text{P-3}: \{6M_{\text{b}}+4, 3M_{\text{b}}+5, M_{\text{c}}\}, \\ \text{P-4}: \{3M_{\text{c}}M_{\text{b}}+5M_{\text{c}}+3M_{\text{b}}+2, 3, M_{\text{b}}\}, \\ \text{P-5}: \{3M_{\text{c}}M_{\text{b}}+5M_{\text{c}}+6M_{\text{b}}+3, 2, 2\}, \\ \text{P-6}: \{3M_{\text{c}}M_{\text{b}}+5M_{\text{c}}+6M_{\text{b}}+8, 3, M_{\text{b}}\}, \end{array} \right. \tag{11}$$

with

$$\left\{ \begin{array}{l} M_{\text{b}} = \lfloor \dfrac{M_t - 5}{6} \rfloor, M_t \geq 11, \\ M_{\text{c}} = M_{\text{t}} - 3M_{\text{b}} - 4, \end{array} \right. \tag{12}$$

where M_t is the number of sensors in transmit array and $\text{P}-i$ are ULAs. In Fig. 2, the method of using UF-3BL to design sparse MIMO array based on our strategy is presented, where $S_{\text{t}} = Z^{\max}_{\text{UF-3BL}}$ is the unit element spacing for the receive array. Based on (9), one should first find the exact expression for the DCA of UF-3BL in order to obtain the feasible value of $Z^{\max}_{\text{UF-3BL}}$. Based on [14], the DCA of UF-3BL have the following expressions

$$\begin{aligned} P^{\text{DCA}}_{\text{UF-3BL}}(x) = &\underbrace{x^0 + \cdots + x^{8M_{\text{c}}+2}}_{\text{consecutive}} + \underbrace{x^{8M_{\text{c}}+4} + \cdots + x^{8M_{\text{c}}+7}}_{\text{consecutive}} \\ &+ \underbrace{x^{8M_{\text{c}}+9} + \cdots + x^{8M_{\text{c}}+11}}_{\text{consecutive}} + x^{8M_{\text{c}}+14}, \end{aligned} \tag{13}$$

when $M_{\text{b}} = 1$, and

$$\begin{aligned} P^{\text{DCA}}_{\text{UF-3BL}}(x) = &\underbrace{x^0 + \cdots + x^{3M_{\text{b}}M_{\text{c}}+5M_{\text{c}}+3M_{\text{b}}-1}}_{\text{consecutive}} \\ &+ \underbrace{x^{3M_{\text{b}}M_{\text{c}}+5M_{\text{c}}+3M_{\text{b}}+1} + \cdots + x^{3M_{\text{b}}M_{\text{c}}+5M_{\text{c}}+6M_{\text{b}}+5}}_{\text{consecutive}} \\ &+ x^{3M_{\text{b}}M_{\text{c}}+5M_{\text{c}}+6M_{\text{b}}+8}(x^0 + x^3 + \cdots + x^{3(M_{\text{b}}-1)}), \end{aligned} \tag{14}$$

when $M_{\text{b}} \geq 2$. Based on [14], the dual array of $P^{\text{DCA}}_{\text{UF-3BL}}(x)$ can be given as

$$\begin{aligned} P^{\text{DCA-dual}}_{\text{UF-3BL}}(x) = x^0 + &\underbrace{x^3 + \cdots + x^5}_{\text{consecutive}} + \underbrace{x^7 + \cdots + x^{10}}_{\text{consecutive}} \\ &+ \underbrace{x^{12} + \cdots + x^{8M_{\text{c}}+14}}_{\text{consecutive}}, \end{aligned} \tag{15}$$

when $M_{\text{b}} = 1$ and

$$\begin{aligned} P^{\text{DCA-dual}}_{\text{UF-3BL}}(x) = &(x^0 + x^3 + \cdots + x^{3(M_{\text{b}}-1)}) \\ &+ \underbrace{x^{3M_{\text{b}}} + \cdots + x^{6M_{\text{b}}+4}}_{\text{consecutive}} \\ &+ \underbrace{x^{6M_{\text{b}}+6} + \cdots + x^{3M_{\text{b}}M_{\text{c}}+5M_{\text{c}}+9M_{\text{b}}+5}}_{\text{consecutive}}, \end{aligned} \tag{16}$$

when $M_{\text{b}} \geq 2$.

Then, based on (13), (14), (15) and (16), the solution of (9) can be expressed as

$$\alpha = \left\{ \begin{array}{ll} x^{8M_{\text{c}}+3}, & M_{\text{b}} = 1 \\ x^{3M_{\text{b}}M_{\text{c}}+5M_{\text{t}}+3M_{\text{b}}}, & M_{\text{b}} \geq 2 \end{array} \right. \tag{17}$$

with

$$Z^{\max}_{\text{UF-3BL}} = \left\{ \begin{array}{ll} 16M_{\text{c}} + 17, & M_{\text{b}} = 1 \\ 6M_{\text{b}}M_{\text{c}} + 10M_{\text{c}} + 12M_{\text{b}} + 5. & M_{\text{b}} \geq 2 \end{array} \right. \tag{18}$$

Hence, the unit element spacing for the receive array is $Z_{\text{UF-3BL}}^{\max}$. Based on [15], the closed form expression for ATLI-1BL is given by

$$\begin{cases} \text{P-1}: \{0, 1, M_{\mathrm{a}}\}, \\ \text{P-2}: \{2M_{\mathrm{a}}-1, 2M_{\mathrm{a}}-1, M_{\mathrm{a}}-1\}, \\ \text{P-3}: \{2M_{\mathrm{a}}^2-M_{\mathrm{a}}, 4M_{\mathrm{a}}-1, M_{\mathrm{f}}\}, \\ \text{P-4}: \{4M_{\mathrm{a}}M_{\mathrm{f}}+2M_{\mathrm{a}}^2-M_{\mathrm{f}}-3M_{\mathrm{a}}+1, 2M_{\mathrm{a}}, M_{\mathrm{a}}-1\}, \\ \text{P-5}: \{4M_{\mathrm{a}}M_{\mathrm{f}}+4M_{\mathrm{a}}^2-M_{\mathrm{f}}-5M_{\mathrm{a}}+1, 1, M_{\mathrm{a}}\}, \end{cases} \tag{19}$$

with

$$\begin{cases} M_{\mathrm{a}} = \left\lfloor \dfrac{M_{\mathrm{r}}+4}{6} \right\rfloor, \\ M_{\mathrm{f}} = M_{\mathrm{r}} - 4M_{\mathrm{a}} + 2, \end{cases} \qquad M_{\mathrm{r}} \geq 14. \tag{20}$$

B. uDOF of UF-MIMO and Optimal Parameter Selection

The uDOF of UF3-ATLI1BL MIMO depends on the uDOF of the UF-3BL and the ATLI-1BL. Based on [14], the uDOF of UF-3BL can be expressed as

$$\text{uDOF}_{\text{UF-3BL}} = \begin{cases} \dfrac{M_{\mathrm{t}}^2}{2} + 2M_{\mathrm{t}} - 11 & M_{\mathrm{t}}\%6 = 0, 4, \\ \dfrac{M_{\mathrm{t}}^2}{2} + 2M_{\mathrm{t}} - 9.5 & M_{\mathrm{t}}\%6 = 1, 3, \\ \dfrac{M_{\mathrm{t}}^2}{2} + 2M_{\mathrm{t}} - 9 & M_{\mathrm{t}}\%6 = 2, \\ \dfrac{M_{\mathrm{t}}^2}{2} + 2M_{\mathrm{t}} - 13.5 & M_{\mathrm{t}}\%6 = 5, \end{cases} \tag{21}$$

when $M_{\mathrm{t}} \geq 17$ and

$$\text{uDOF}_{\text{UF-3BL}} = 16M_{\mathrm{t}} - 107, \tag{22}$$

when $17 > M_{\mathrm{t}} \geq 11$.

Based on [15], the uDOF of ATLI-1BL can be expressed as

$$\text{uDOF}_{\text{ATLI-1BL}} = \begin{cases} \dfrac{2M_{\mathrm{r}}^2 + 2M_{\mathrm{r}}}{3} - 3 & M_{\mathrm{r}}\%6 = 0, 2, \\ \dfrac{2M_{\mathrm{r}}^2 + 2M_{\mathrm{r}}}{3} - 1 & M_{\mathrm{r}}\%6 = 1, 5, \\ \dfrac{2M_{\mathrm{r}}^2 + 2M_{\mathrm{r}} - 1}{3} & M_{\mathrm{r}}\%6 = 2, 4, \\ \dfrac{2M_{\mathrm{r}}^2 + 2M_{\mathrm{r}} - 4}{3} - 5 & M_{\mathrm{r}}\%6 = 3, \end{cases} \tag{23}$$

when $M_{\mathrm{r}} \geq 14$ and

$$\text{uDOF}_{\text{ATLI-1BL}} = 14M_{\mathrm{r}} - 67, \tag{24}$$

when $14 > M_{\mathrm{t}} \geq 7$.

Based on (21), (22), (23) and (24), the optimal parameter selection for UF3-ATLI1BL MIMO can be written as

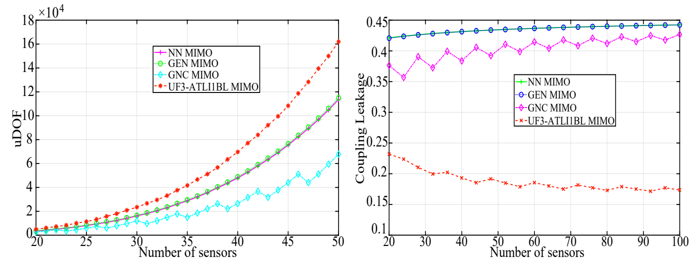
$$M_{\mathrm{t}} = 11, \tag{25}$$

when $22 > M \geq 18$ and

$$M_{\mathrm{t}} = \begin{cases} 12 & M = 22, 23, 26, \\ 13 & M = 24, 26, 27, 28, \\ 14 & M = 29, 30, \\ 15 & M = 31, 32, \end{cases} \tag{26}$$

when $33 > M \geq 22$ and

$$M_{\mathrm{t}} = \begin{cases} \left\lfloor \dfrac{M}{2} \right\rfloor & M\%12 = 0, 1, 9, 10, 11, \\ \left\lfloor \dfrac{M}{2} \right\rfloor - 1 & M\%12 = 2, 3, 4, 5, 6, 7, 8, \end{cases} \tag{27}$$



(a) uDOF.

(b) Coupling leakage.

Fig. 3: uDOF and coupling leakage versus number of sensors.

when $105 > M \geq 33$, where $\lfloor \cdot \rfloor$ indicates floor operation, and

$$M_{\mathrm{t}} = \begin{cases} \left\lfloor \dfrac{M}{2} \right\rfloor & M\%12 = 0, 10, \\ \left\lfloor \dfrac{M}{2} \right\rfloor - 1 & M\%12 = 1, 2, 3, 4, 5, 6, 7, 8, \\ \left\lfloor \dfrac{M}{2} \right\rfloor + 1 & M\%12 = 9, 11, \end{cases} \tag{28}$$

when $129 > M \geq 105$ and

$$M_{\mathrm{t}} = \begin{cases} \left\lfloor \dfrac{M}{2} \right\rfloor & M\%12 = 0, 1, 10, \\ \left\lfloor \dfrac{M}{2} \right\rfloor - 1 & M\%12 = 2, 3, 4, 5, 6, 7, 8, 9, \\ \left\lfloor \dfrac{M}{2} \right\rfloor + 1 & M\%12 = 11, \end{cases} \tag{29}$$

when $M \geq 129$.

The final uDOF for UF3-ATLI1BL MIMO can be given as

$$\text{uDOF} = Z_{\text{UF-3BL}}^{\max}(\text{uDOF}_{\text{ATLI-1BL}} - 1) + \text{uDOF}_{\text{UF-3BL}} - 2, \tag{30}$$

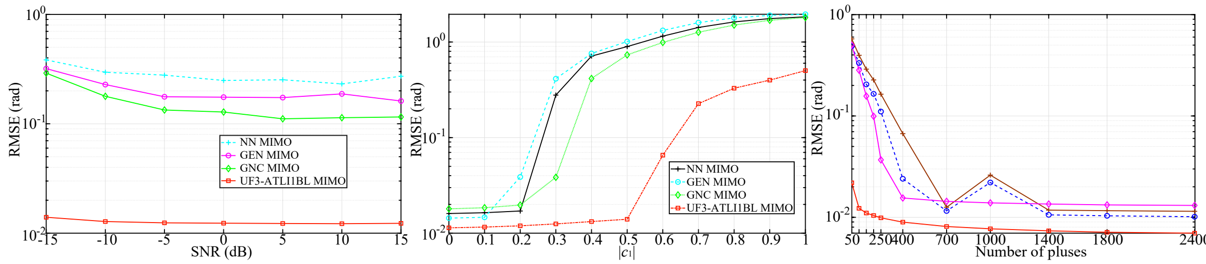
where parameters can be found in the aforementioned equations.

IV. Simulations

In this section we present the simulation results of the presented UF3-ATLI1BL MIMO and compare them with the state-of-the-art results (NN-MIMO [10], GNC-MIMO with 3 sub-ULAs in receive array [12], GEN-MIMO [13]). The number of radar pulses used is 500. For all the SA geometries tested, DOAs are computed based on the spatial smoothing MUSIC algorithm [7], [17].

A. uDOF and Coupling Leakage

The uDOF and coupling leakage (see (12) in [18]) with respect to the number of sensors are presented in Fig. 3. With an increasing number of sensors, the coupling leakage of NN-MIMO, GNC-MIMO and GEN-MIMO increases, which indicates the mutual coupling among sensors is increased. On the contrary, the proposed UF3-ATLI1BL MIMO has much lower coupling leakage than other sparse MIMO arrays tested, which means the mutual coupling is reduced significantly. Besides, the uDOF of the proposed UF3-ATLI1BL MIMO is much higher than the relevant configurations.



(a) RMSE vs. SNR. (b) RMSE vs. $|c_1|$ (see (9) in [18]). (c) RMSE vs. number of pulses.

Fig. 4: RMSE performance for configurations tested in different conditions.

B. MIMO Array Performance Considering Coupling

The RMSE performance of the four MIMO structures are compared in Fig. 4, where all structures have 20 sensors. For NN-MIMO and GEN-MIMO, $M_{\mathrm{t}} = M_{\mathrm{r}} = 10$ and for the proposed UF3-ATLI1BL MIMO, $M_{\mathrm{t}} = 11, M_{\mathrm{r}} = 9$, while for the GNC-MIMO, $M_{\mathrm{t}} = 5, M_{\mathrm{r}} = 15$.

The RMSE versus SNR is presented in Fig. 4 (a), where we observe 50 uniformly located targets in $[-60^\circ, 60^\circ]$ and $c_1 = 0.3e^{j\pi/3}$, $c_i = c_1 e^{-j(i-1)\pi/8}/i$, $i = 2, \dots, 100$. In this case, one can see that the proposed UF3-ATLI1BL MIMO has a significant improvement for all SNR values, whereas NN-MIMO, GEN-MIMO, and GNC-MIMO all face performance degradations.

It is also necessary to test the performance of relevant SA structures in various mutual coupling scenarios. We keep SNR=0 dB and vary the coupling coefficient from $|c_1| = 0$ to $|c_1| = 1$, where the sources are the same as in the last case. It can be found from Fig. 4 (b) that the proposed UF3-ATLI1BL MIMO can provide excellent performance in both low and high coupling environments. In particular, UF3-ATLI1BL MIMO shows a good improvement compared to existing MIMO SAs in high coupling environment.

In the last case, the RMSE performance with respect to the number of radar pulses in high coupling environment is investigated. In this case, 30 targets are uniformly located between -70° to 50° and the coupling parameters are set as $c_1 = 0.3e^{j\pi/3}$, $c_i = c_1 e^{-j(i-1)\pi/8}/i$, $i = 2, \dots, 100$. From the results shown in Fig. 4 (c), it can be concluded that the UF3-ATLI1BL MIMO has the best performance. It is also worthy to note that with fewer radar pulses, UF3-ATLI1BL MIMO has a considerable advantage in comparison with other existing sparse MIMO arrays.

V. Conclusion

In this letter, the UF principle is considered for designing sparse MIMO array with desired property and a novel strategy is proposed accordingly. The proposed strategy can make full use of all the sensors in the DCSC domain and significantly improve the final uDOF. Based on the proposed strategy, the sparse MIMO array design problem becomes equivalent to solving a polynomial equation. Finally, a novel sparse MIMO array is devised using UF idea with not only low mutual coupling but also high uDOF. Simulation results verify the advantages of the proposed design and the effectiveness of the proposed strategy.

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