

Forecasting Buoy Observations Using Physics-Informed Neural Networks

Austin B. Schmidt , Pujan Pokhrel , Mahdi Abdelguerfi , Elias Ioup , and David Dobson 

Abstract—Methodologies inspired by physics-informed neural networks (PINNs) were used to forecast observations recorded by stationary ocean buoys. We combined buoy observations with numerical models to train surrogate deep learning networks that performed better than with either data alone. Numerical model outputs were collected from two sources for training and regularization: the hybrid circulation ocean model and the fifth ECMWF reanalysis experiment. A hyperparameter determines the ratio of observational and modeled data to be used in the training procedure, so we conducted a grid search to find the most performant ratio. Overall, the technique improved the general forecast performance compared with nonregularized models. Under specific circumstances, the regularization mechanism enabled the PINN models to be more accurate than the numerical models. This demonstrates the utility of combining various climate models and sensor observations to improve surrogate modeling.

Index Terms—Deep learning, ECMWF re-analysis v5 (ERA5), hybrid circulation ocean model (HYCOM), physics-informed neural network (PINN), recurrent model, surrogate model.

I. INTRODUCTION

OCEAN parameter forecasting is studied for various applications, such as climate modeling, marine life population surveying, and water quality monitoring. There is a clear need across industries to have fast and far-reaching forecasts. As such, research and improvements in ocean and climate modeling tools have continued to be interesting and necessary in literature. Well-studied numerical solutions for this task include Navier–Stokes and advection–diffusion, which are formulated as sets of partial differential equations (PDEs) for modeling flow systems. Building primitive equations into a more complex model yields global ocean and climate models for accurate, full-coverage simulations [1], [2], [3]. The initial values and boundary conditions of the modeled system are important for accurately modeling physical behaviors in this way [4]. Initial

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values are recorded as sparse observations across the world’s oceans using different methods. These methods include free-floating buoys that record data by following ocean currents, stationary buoys for monitoring fixed locations, and satellites for collecting global imagery [5]. As the viability of the modeled forecasts greatly depends on accurate estimations of the initial values, data assimilative systems have been a point of research, and assimilating observations with numerical models has shown improved results [6]. In the case of the United States Navy, researchers have developed the global coupled atmosphere–ocean–sea ice forecasting system called the Navy Earth System Prediction Capability where modeled data are assimilated with observations for an improved result [7], [8]. However, observations can be missing such that there is no data availability. In this situation, the data assimilation scheme cannot be taken advantage of. Therefore, there exists some motivation to generate discrete observation forecasts for their integration into an assimilation pipeline. To this end, we investigate a generalized procedure to predict sparse ocean observation values.

Surrogate deep learning models are trained using available historical data to model a system given prior input values. The main benefit of this technique is that forecasts are generated more quickly than when evolving a numerical model. Recurrent neural network (RNN) architectures, such as long short-term memory (LSTM) networks and Transformers, are used to propagate information forward when making long-term predictions, making them popular choices for modeling ocean parameters as surrogate models [5]. When surrogate modeling ocean parameters, data are required from recorded observations, numerical model outputs, or both. In this work, we take particular interest in two data assimilated numerical models, which provide training and regularization data. The hybrid circulation ocean model (HYCOM) is a hybrid isopycnic model, which sees improvement over its predecessor in shallow water and unstratified ocean regions [1]. ECMWF re-analysis v5 (ERA5) is the fifth reanalysis experiment of the European Centre for Medium-Range Weather Forecasts (ECMWF) model for global climate and weather features [2].

By combining the numerical models with buoy-collected observation data, we show how a physics-regularized approach can be used to improve observation forecasting. Thus, we consider physics-informed neural networks (PINNs) for approximating numerical models to accurately forecast a single discrete point (i.e., an observation). A PINN is a neural network, which is regularized at training time by applying penalties in the loss function. The penalties are scored by comparing adherence

to a PDE-based numerical model [4]. We investigate if the forecasting result of real-world sensor data collected by stationary ocean buoys can be more accurately forecasted when regularized by the prior mentioned numerical models. Since reanalysis data exists for many ocean and climate features, we use the high-quality numerical model outputs to regularize our PINN model.

As far as we know, we are the first to integrate HYCOM and ERA5 data as a regularizing source in a PINN-inspired network. We show that the physical models may be used with recorded buoy data to provide more stable long-term predictions due to the regularization support. Our methodology differs from other PINN research by modeling only observations and, more importantly, by the way in which we implement the loss function. These differences will be discussed further in the upcoming related works section. To assess our models, sea surface temperature (SST), gust strength, and air pressure are sparsely forecasted using our technique. The main contributions of this article are as follows. We train deep learning models to recursively forecast physical parameters as recorded by free-floating ocean buoys. We define a custom loss function to use numerically modeled data and observation data as sources for training physics regularized models. The methodology is capable of handling situations where a physical parameter is available from both sources or a single source. When both sources of data are available for a feature, we show how the surrogate may be trained using a ratio of the training errors from each source. The most performant surrogate for the test data are found through a grid search of the static regularization term, λ , which controls the ratio of errors. We demonstrate the flexibility of PINNs to combine different numerical models using a surrogate deep learning model, which outperforms the nonregularized deep learning models. We discuss the numerical models and their effect on the rolling forecast ability of our surrogate model for up to 24 h.

The rest of this article is organized as follows. Section II presents the related works. Section III presents the methods used in this article. Section IV presents the results. Finally, Section V concludes this article.

II. RELATED WORKS

Ocean surrogate models have been advancing with the advent of deep learning, and more refined machine learning approaches [5]. Research into deep learning surrogate modeling of SST shows promising results as SST can be forecasted as discrete points [9], as a field [10], or as a super-resolution field [11]. Instead of directly solving intractable formulations, such as Navier–Stokes or other prognostic equations, for ocean modeling, a data-driven surrogate model is trained using the substantial amounts of historical training data available via numerical models or raw observations [12]. The use of observation assimilated models to train deep learning surrogates has been seen multiple times using both HYCOM [13], [14] and ERA5 [15], [16], [17] models. Through back propagation a deep learning model learns a parameterized representation of the underlying physical phenomenon, which are otherwise modeled numerically. Surrogate models may be preferred over traditional

models due to faster outputs once the model has been trained [8]. For example, in [18], approaching hurricane parameters are forecasted in seconds. Machine learning surrogate models will generally have more numerical instability when compared with numerical models in forecasting experiments. This speed and accuracy tradeoff is seen in the conclusions of surrogate modeling studies for data assimilation in dynamic subsurface flow [12] and regional wind/wave forecasting [19]. In both papers, the forecast accuracy was similar or lower than numerical models, but the computational speed was greatly improved. One keynote on numerical stability and model accuracy is that the generalization of machine learning surrogate modeling is not assured for all cases. Authors observe the stability difference in operational planning with dynamic constraints where the forecasting stability is very good for some deep learning surrogate models but unstable when using other machine learning techniques [20]. This forecasting stability problem is also considered in [21], where outputs of physics-based numerical models are combined and used as supervised learning training sets to promote more accurate forecasts than when used independently. Furthermore, the surrogate modeling task can be used with data assimilation to correct numerical model error in an online fashion [22]. As such, surrogate models have a place among the more carefully calculated simulation-based numerical models, such as HYCOM and ERA5. This is especially true in applications where numerical solutions are too complex or computationally intensive for real time analysis and the acceptable error threshold is high.

PINNs are referred to as such because they leverage physical constraints within the model's loss function during training to enforce convergence to governing physical laws. This type of network was popularized in the deep learning community by Raissi et al. [23] in 2017 and 2019. The introduction of differential equations that define physical phenomenon to the training procedure is found to improve the model's resilience to noise [24]. PINNs are regularized in training by comparing model performance to the adherence of the introduced PDEs while also fitting data points to unique solutions [25]. The result of these forecasting models is that we can incorporate noisy data into existing algorithms, ignore complex mesh generation, and tackle high-dimensional problems governed by parameterized PDEs. Originally, research has focused on surrogate modeling with PINNs for solving systems governed by the Burgers' and Navier–Stokes equations [26]. PINNs have recently been investigated in industry informatics settings, such as modeling flow equations for ocean models [24], modeling crack propagation [27], [28], modeling leakage [29], modeling faults [30], and modeling electric loads [31]. Forecasting SST is commonly found as a full-coverage modeling problem combining either generative models [32], [33] or convolutional neural networks [34] with various PDEs. Continual discussion on PINNs and the types of equations usually solved can be reviewed in [4] and [35].

We have not seen any other works that use a ratio of numerical model data and observations to train and regularize a deep neural network for surrogate modeling. Our methods share similarities with [21], who utilizes numerical models as

training data for surrogate models. However, we employ our PINN-inspired approach to regularize models by combining both observations and numerical outputs. Furthermore, our work differs methodologically from the prior mentioned PINN research in two significant ways. First, there is no differentiation or simulation step to solve selected PDEs within the surrogate training procedure. This is the case because the numerical model pipeline is too computationally intensive for this to be feasible. Instead, the selected climate and oceanography models, HYCOM and ERA5, have already undergone comprehensive modeling and data assimilation processes, which provide high quality, historical simulation data. Using the precomputed data instead of directly solving PDEs means the numerical model can be arbitrarily complex and we do not need to implement the formulation for use in our framework. The second divergence is the role of the hyperparameter λ within the PINN loss function. The traditional PINN training loss function sums the performance of the surrogate model and the divergence when compared with the numerical solution of selected PDEs. In that case, λ is used as the multiplicative weighting term to determine how much of a contribution the divergence from the numerical solution has on the final loss output. Instead, we use λ as a mechanism to control a weighted ratio of observation versus modeled data in training. This ratio of loss from multiple sources improves the training process when numerical data, observational data, or both are noisy. The proposed buoy forecasting task is inspired by Pokhrel et al. [36], but we forecast multiple buoy parameters, test additional numerical models (ERA5 and HYCOM), and apply our physics-regularized training methodology, as main differences. So, we show, in an experimental approach, that we may use complex solutions calculated by numerical climatology and ocean flow models as a means of regularizing surrogate PINN models. We aim to demonstrate that a PINN can internalize the simulated outputs of ocean and climate models to be more capable of forecasting unseen buoy values.

III. METHODS

In this section, we discuss the methodologies utilized in investigating our PINN-inspired surrogate models. The models are trained to forecast ocean observations at fixed locations given prior conditions. The numerical models, HYCOM and ERA5, regularize the model at training time and offer additional input features. Section III is organized as follows. Section III-A defines the numerical models overview; Section III-B presents the data and feature processing; Section III-C presents deep learning models; and Section III-D presents metrics and testing strategy.

A. Numerical Models Overview

The HYCOM system is a primitive equation model for general ocean circulation that evolved from the Miami Isopycnic-Coordinate Ocean Model (MICOM) system developed by Bleck in [1] and Halliwell [3]. HYCOM, such as MICOM, is a primitive-equation model containing five prognostic equations. Two equations for the horizontal velocity components, a mass

continuity or layer thickness tendency equation, and two conservation equations for a pair of thermodynamic variables, such as salt and temperature or salt and density. The authors also define several diagnostic equations to control the spacing and movement of layer interfaces. This includes the hydrostatic equation, which links temperature, salinity, and pressure, alongside an equation prescribing the vertical mass flux through a surface. A hybrid grid-generating technique determines whether isopycnal or inflated nonisopycnal layers are specified [1]. Beyond the general governing equations and gridding algorithm, HYCOM has specialized mixing processes, many of which are shared with the MICOM implementation. Temperature and salinity profiles are assimilated into the ocean flow model to improve initial analysis. The specific HYCOM implementation we use for data is the 41-layer HYCOM + NCODA Global 1/12° reanalysis experiment.

ERA5 is the fifth ECMWF reanalysis for global climate and weather features. The atmospheric global reanalysis (HRES) includes the period from January 1950 to the present year. ERA5 reanalysis is produced using the 4D-Var data assimilation technique and model forecasts with 137 hybrid vertical sigma/pressure levels [2]. The data assimilation of ERA5 also contains an ensemble system (EDA) of ten members for providing background error estimates. The model assimilates as many observations as possible in the upper air and near-surface regions. This forecasting system includes over a decade of research and development for all components: atmosphere, land, and ocean waves. The integrated forecast system (IFS) implemented by ECMWF has its equations expertly discussed in the documentation manual [37] and is more generally discussed in [2]. We specifically use the ERA5 hourly data on single levels from 1959 to the present [38], which is a data assimilative reanalysis that uses the 2016 version of the ECMWF numerical weather prediction model and data assimilation system (IFS Cy41r2). The ERA5 implementation is modeled at 1/4° latitude/longitude increments. Thus, the resolution of ERA5 is lower than that of HYCOM.

Given these arbitrarily complex numerical models, which are precomputed, we do not need to implement the PDEs, which govern the models directly. Instead, we will use the outputs from both models as training and regularization data within our deep learning models. To yield discrete value forecasting in a generic manner, we only need the values, which are geographically closest to the latitude and longitude of the buoy observations. Likewise, we collect the discrete time step temporally closest to the observations we are interested in. Therefore, we consider a generic method for retrieving data from full-coverage numerical models in

$$f_m(t, x, y) = v. \quad (1)$$

For a sufficiently complex model f_m , we input the desired period t and the closest possible latitude and longitude, x and y . This yields whichever set of scalar features v are desired from the numerical model. These values can then be used as regularization data, training data, or both for a deep learning PINN model. This formulation is useful in our methodology where we want to train a neural network on the observations

304 themselves while regularizing with numerical model data. This
 305 differs to similar PINNs that provide full-coverage modeling of
 306 ocean and climate features, where the training data are limited
 307 to full-coverage reanalysis and the regularizing PDEs are for-
 308 mulated from simpler equations as seen in [32], [33], and [34].

309 B. Data and Feature Processing

310 Both buoy observations and numerical model outputs are pub-
 311 licly available and have decades worth of data. In this study, we
 312 select dates from January 1st, 2011 to December 31st, 2011. The
 313 buoy data, which comprises the observation data for this study,
 314 comes from 3-m discus Self-Contained Ocean Observations
 315 Payload sensor package buoys and Waverider buoys. We select
 316 124 candidate buoys from around the United States East and
 317 West Coasts, the Caribbean, and the Gulf of Mexico. The buoy
 318 data are collected from the National Oceanic and Atmospheric
 319 Administration (NOAA) public data center. NOAA arranges
 320 individual buoys systematically by assigning each one a distinct
 321 identification (ID) number. The specific ID corresponding to
 322 each buoy selected for analysis is found in the Appendix. Water
 323 temperature, air pressure, and gust strength are extracted from
 324 the buoy feature set to provide the real-world recorded result.
 325 Since HYCOM and ERA5 are both gridded datasets, we select
 326 the data points which match the latitude and longitude as closely
 327 as possible to each buoy position. HYCOM snapshots are taken
 328 every 3 h, and most buoys are recorded at the 50th min of each
 329 hour. Therefore, we forecast buoy features in 3-h increments. To
 330 facilitate the coupling of the numerical models and buoy data,
 331 we select buoy features that have matching modeled numerical
 332 features. Out of the 18 selected features, water temperature, gust
 333 strength, and air pressure are shared by the numerical models and
 334 the buoys, so they will be coupled in training time, as described
 335 by the loss function. We display all features recorded from the
 336 buoys and numerical models in Table I along with their original
 337 units.

338 It is possible that data are missing from our data sources in
 339 two separate ways. A value may be missing temporally such
 340 that no data are recorded at all for a particular time step. This
 341 is most common in the NOAA buoy data where, for example, a
 342 buoy faces mechanical failure and cannot record observations for
 343 days to months at a time. Therefore, our training and testing data
 344 are limited by the amount of available buoy-recorded data. The
 345 numerical models do not leave a time step without data except
 346 in one case, a 24 h gap found within the HYCOM dataset. Since
 347 this represents only eight data points, we cover the temporal
 348 gap by replacing the missing time steps with the previous 24 h
 349 period. Otherwise, for a given time step, features may be missing
 350 data and are replaced with fill values of 99, 999, 9999, or
 351 $-32\ 767$, depending on the data source and feature. Each of
 352 our sources of data exhibits at least some fill data, depending
 353 on the geographical region or time of year. We remove all fill
 354 values from the data and, in their place, linearly interpolate
 355 the missing values forward and backward for that individual
 356 buoy or numerical model. If any numerical model data source
 357 are composed of more than 20% fill values, we disregard that
 358 corresponding buoy from the training and testing pipeline. No
 359 buoys are discarded for having too many fill values for the

TABLE I
DATA FEATURES AND THEIR SOURCES

Feature Name	Feature Units	Feature Source
Water Temperature	°C	Buoy
Gust Strength	m/s	Buoy
Air Pressure	hPa	Buoy
Water Temperature	°C	HYCOM
Salinity	psu	HYCOM
Surf Elevation	m	HYCOM
Water Eastern Flow (U)	m/s	HYCOM
Water Northern Flow (V)	m/s	HYCOM
Wind Eastern Flow (U)	m/s	ERA5
Wind Northern Flow (V)	m/s	ERA5
Evaporation	m of w.e.	ERA5
Gust Strength	m/s	ERA5
Mean evaporation Rate	kg/(m ⁻² s ⁻¹)	ERA5
Mean Runoff Rate	kg/(m ⁻² s ⁻¹)	ERA5
Sea-Ice Cover (%)	[0-1]	ERA5
Air Pressure	hPa	ERA5
Cloud Cover	[0-1]	ERA5
Precipitation	m	ERA5

In bold are numerical model features to be coupled as a regularization mechanism when forecasting buoy observations.

360 purpose of preserving as much data for training and testing as
 361 possible. It is important to note that the retention of buoys with
 362 interpolated values can have an impact on model accuracy.

363 The processed data are split into three datasets for training,
 364 validation, and testing. As each buoy is missing various days,
 365 we select the train, test, and validation splits by date. Therefore,
 366 all members of the training data are chosen from January 1st
 367 to September 13th. The validation data is from September 13th
 368 to October 20th. The testing data includes the remainder of the
 369 year. Since the buoys are missing data at separate times of the
 370 year, a buoy may occasionally contribute to one dataset but not
 371 another. We specify the buoy selection in Table VI where we
 372 display the number of buoys allowed into each dataset. There
 373 are 148 365 training instances, 23 118 validation instances, and
 374 48 039 testing instances. Among the original 124 buoys selected
 375 for processing, only 86 buoys had training, validation, and
 376 testing data available. Each feature is independently normalized
 377 between -1 and 1 before training, using the training data min-
 378 imum and maximum values. This approach is essential in deep
 379 learning to prevent data with varying scales from dominating
 380 the network's performance. As our network is trained on scaled
 381 data, we transform the network's output to its original scale for
 382 meaningful result comparison.

383 To understand the impact of first-order differenced data on our
 384 regularizing technique, we studied two separate setups. In the
 385 first, we train the models using the original values recorded by
 386 the data sources. Subsequently, we take the first-order difference
 387 to train the model on the differences between time steps. Training
 388 with differenced values to make the data stationary is seen for

nonregularized RNNs [39] and physics regularized RNNs [40] when forecasting time series. Stationarity means that a time series has been stabilized such that it has consistent statistical properties, such as mean and variance [41]. Nonstationary data contains trends and seasonality that may introduce bias to the surrogate models. Taking the first-order difference of our data removes trends in the training data and makes the analysis problem more forgiving. The result is that modeling using the differenced data will result in higher accuracy and a more stable forecast. The more consistent statistics also imply more accurate scaling when normalizing the test data. Nonstationary data are still useful for models with longer context windows or the addition of features, which are embedded in time, so testing both data representations is worthwhile. In our experiments, we will clearly denote the data used when training or evaluating a surrogate model as either original data or differenced data. When comparing models which forecast the differences in data rather than the original data, we need to transform the resulting forecast back to the original scale. This transformation is computed by summing the forecast f_t with the initial conditions x_{t-1} , then that value is summed iteratively with each following difference forecast in the horizon window.

411 C. Deep Learning Models

412 A PINN is made up of any general network architecture. Since
413 we are forecasting time series, we experiment on architectures
414 that utilize gated recurrent units (GRU) units, LSTM units, and
415 Transformer units. Layers of these units are accompanied by
416 dense fully connected layers, normalization layers, and training
417 dropout layers. Each layer includes a nonlinear activation
418 function except for some dense layers, which are linear in the
419 Transformer architecture. Between the layers, we add dropout
420 layers with 5% dropout rate during training for the Transformer
421 and 10% for the LSTM. Similarly, we apply a normalization
422 layer in between dense and LSTM layers to prevent exploding
423 or vanishing gradients. The Transformer block is made of ten
424 attention heads. The exact summary of the LSTM-based and
425 Transformer-based models can be seen in Tables II and III.
426 The GRU-based model architecture is the same as the LSTM
427 model. The number of trainable parameters is lesser for the
428 GRU compared with the LSTM but is otherwise the same
429 structure. The GRU and LSTM models have much fewer weights
430 than the Transformer-based model, which takes longer to train.
431 We include each layer of the model, the number of trainable
432 parameters, and the activation at that layer, if any. The GRU and
433 LSTM models are trained for 100 epochs while the Transformer
434 model is trained for 200 epochs, due to the increased number of
435 trainable weights. A data batch size of 256 was used in all cases.
436 To optimize the value in each epoch of back-propagation, the
437 Adam optimizer is selected for the Transformer model and root
438 mean square propagation (RMSProp) for the LSTM and GRU
439 networks. The models are always trained using the same random
440 seed to ensure experiments are as uniform as possible.

441 Each model, once initialized, is trained to accept the 18
442 specified features as input and produce the predicted next step
443 for each feature as output. Since each model is trained to produce

TABLE II
LSTM MODEL ARCHITECTURE

Layer Type	Output Shape	Param #	Activation
Input Layer	(N, 18, 1)	0	None
Reshape	(N, 1, 18)	0	None
Dense	(N, 1, 256)	4864	Tanh
Batch Normalization	(N, 1, 256)	1024	None
Dropout	(N, 1, 256)	0	None
LSTM	(N, 1, 256)	525312	Tanh
Dropout	(N, 1, 256)	0	None
LSTM	(N, 1, 256)	525312	Tanh
Dense	(N, 1, 256)	65792	Tanh
Batch Normalization	(N, 1, 256)	1024	None
Dropout	(N, 1, 256)	0	None
LSTM	(N, 1, 256)	525312	Tanh
Dropout	(N, 1, 256)	0	None
LSTM	(N, 256)	525312	Tanh
Dropout	(N, 256)	0	None
Dense	(N, 200)	51400	Tanh
Dropout	(N, 200)	0	None
Dense	(N, 200)	40200	Tanh
Dropout	(N, 200)	0	None
Dense	(N, 200)	40200	Tanh
Dropout	(N, 200)	0	None
Dense	(N, 200)	40200	Tanh
Dropout	(N, 200)	0	None
Dense	(N, 18)	3618	Tanh

There are 24 total layers with 2 348 546 trainable parameters. N represents a variable batch size.

the same outputs it requires as inputs, this is considered a rolling
forecast model. In this approach, to forecast further into the
future, we may use the model's own outputs from time t as inputs
for forecasting time $t + 1$. This forecasting technique depends
on accurate initial values. Only the first forecast in a period,
 t_0 , is provided with initial conditions, and as time progresses,
inherent chaos or model error will compound within forecasts.
This method yields models, which are not constrained to a single
forecast horizon. Instead, the models are more flexible, and
can generically forecast any number of desired periods, once
provided initial values. Using the numerical model data as inputs
to our deep learning models may be considered self-fulfilling
because reanalysis data includes high-quality features assimilated
with ground truths not yet observed. We point out that the
assimilated data and observations are only used in training time
and when seeding initial values into the model. The subsequent
predictions use the results from the previous prediction cycle.
All else is kept equal among the models, so we may measure the
effects of our methodology across multiple experiments.

To train the models, the loss function for our PINN is designed
such that the outputs from numerical models are coupled with

TABLE III
TRANSFORMER MODEL ARCHITECTURE.

Layer Type	Output Shape	Param #	Activation
Input Layer	(N, 18, 1)	0	None
Reshape	(N, 1, 18)	0	None
Dense	(N, 1, 512)	9728	Linear
Batch Normalization	(N, 1, 512)	2048	None
Transformer Block	(N, 1, 512)	11016692	Selu
Dropout	(N, 1, 512)	0	None
LSTM	(N, 1, 512)	2099200	Tanh
Dropout	(N, 1, 512)	0	None
Dense	(N, 1, 512)	262656	Linear
Dropout	(N, 1, 512)	0	None
Batch Normalization	(N, 1, 512)	992	None
Dense	(N, 1, 200)	2048	Linear
Dropout	(N, 1, 200)	0	None
Dense	(N, 1, 200)	102600	Linear
Dropout	(N, 1, 200)	0	None
Dense	(N, 1, 200)	40200	Linear
Dropout	(N, 1, 200)	0	None
Dense	(N, 1, 200)	40200	Linear
Dropout	(N, 1, 200)	0	None
Flatten Layer	(N, 200)	0	None
Dense	(N, 18)	3618	Linear

There are 21 total layers with 13 619 190 total trainable parameters. N represents a variable batch size.

buoy-extracted real-world values. To do this, a weighted ratio term is used to determine how much of the calculated error comes from the residual of buoy observations versus the residual of the HYCOM and ERA5 modeled features. This combination is completed for all coupled buoy features, i.e., water temperature, gust strength, and surface air pressure. Thus, the piece-wise cost can be calculated as follows in (2)–(7):

$$\Delta_1 = |\hat{y}_{\text{obs}} - y_{\text{obs}}| \quad (2)$$

$$\Delta_2 = |\hat{y}_{\text{obs}} - f_m(t, x, y)| \quad (3)$$

$$\Omega_{\text{coupled feature loss}} = \lambda * \Delta_1 + (1 - \lambda) * \Delta_2. \quad (4)$$

The two Δ terms defined in (2) and (3) represent the absolute error between the predicted observation and the observation ground truth followed by the absolute error of the predicted observation and the numerical model output as defined in (1). The two error terms are weighted by λ , as seen in (4). The selected λ value represents a ratio to determine how much weight is provided to each ground truth. This coupled feature loss is only calculated for those features, which have both an observational and modeled collection of data available. Through additional feature collection, the technique can be extended to couple any number of observation features to numeric models

$$\Omega_{\text{modeled feature loss}} = |\hat{y}_{\text{model}} - f_m(t, x, y)| \quad (5)$$

$$\Omega_{\text{observed feature loss}} = |\hat{y}_{\text{obs}} - y_{\text{obs}}|. \quad (6)$$

The remaining uncoupled features, as seen in (5) and (6), are used to collect loss in a more traditional way. Excluding the coupled features from the calculation, numerical feature forecasts are measured against numerical model values only and forecasted observational data are measured against observational ground truth only. We include additional numerical features in our setup, which were identified in Table I. There do not exist any noncoupled observational features, so $\Omega_{\text{observation forecast loss}} = 0$, in this experiment. There is no λ controlling the coupling ratio in the case of (5) and (6). The final loss function which combines the disparate loss calculations can be as follows:

$$\Omega_{\text{total loss}} = \Omega_{\text{coupled forecast loss}} + \Omega_{\text{numeric forecast loss}} + \Omega_{\text{observation forecast loss}}. \quad (7)$$

The addition of a coupled loss component is rationalized by considering that as the λ value approach 0.0, we are training our model to behave more like the numerical model, $f_m(t, x, y)$. Conversely, as the λ values approach 1.0, we are promoting forecasts, which more closely resemble the observations, y_{obs} . Expanding the example, when $\lambda = 0.5$, the model balances agreement between both sources equally. In our experiments, the ground truth is measured using y_{obs} , so when $\lambda = 1.0$, we are essentially training a model while using no regularization strategy.

D. Metrics and Testing Strategy

For the original data and differenced data setups the SST, gust strength, and air pressure are forecasted over the reserved testing data for final evaluations of each model. Test horizon windows are conducted from one period to eight periods, where an individual period measures data collected every 3 h. Therefore, this manifests as a one-step three hour forecast through an eight-step 24 h forecast since each forecast step is 3 h apart. Using the rolling forecast property mentioned, we record the mean absolute error (MAE) and root mean square error (RMSE) for each forecast period. The MAE is calculated as follows for an individual buoy $\frac{1}{N} \sum_{i=1}^N (|Y_i^p - Y_i^t|)$, where N is the total number of time steps forecasted, Y^p is the collection of predicted ocean features, and Y^t is the collection of ground truth ocean observations. Similarly, the RMSE is computed as $\sqrt{\frac{1}{N} \sum_{i=1}^N ((Y_i^p - Y_i^t)^2)}$. In analysis, the total MAE and RMSE from our test results are collected from each buoy and then averaged to find the global mean metrics. The best possible model will provide low-value metrics for all forecast periods and features. To verify whether the coupled loss component works as a regularization mechanism, we evaluate for λ values between 0.0 and 1.0 with 0.1 step intervals. Next, we evaluate around the best scoring λ values using 0.02 step intervals. The results gathered in this way may be contrasted with the numerical model outputs from HYCOM and ERA5, which are scored using the same metrics. Using this grid search technique, we are not guaranteed to find the λ value, which yields global minimal error, so we aim to highlight two behaviors instead. The first is that there exists a value of λ , where the RMSE, MAE, or both are

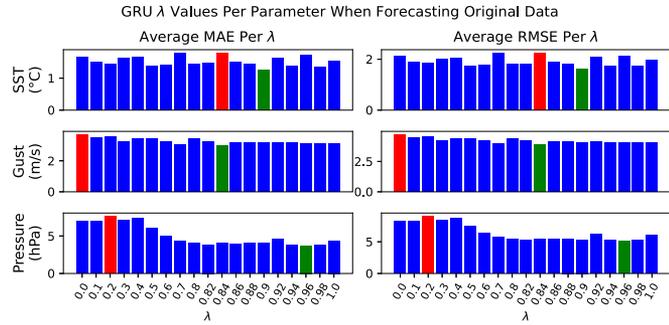


Fig. 1. MAE and RMSE for GRU forecasts from $\lambda = 0.0$ to $\lambda = 1.0$ (no regularization). The lowest scoring λ value is displayed in green while the highest is red. Forecasts are given as the original values.

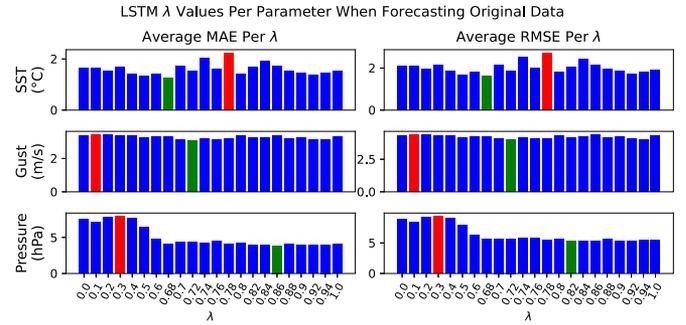


Fig. 2. MAE and RMSE for LSTM forecasts from $\lambda = 0.0$ to $\lambda = 1.0$ (no regularization). The lowest scoring λ value is displayed in green while the highest is red. Forecasts are given as the original values.

533 lesser than $\lambda = 1.0$ (no regularization), for at least one feature
 534 per model. The second is that the selection of best λ is influenced
 535 by inconsistencies in the observation data, misalignment in the
 536 numerical model data, and the PINN architecture.

537 IV. RESULTS

538 We consider which experiments yield the lowest error metrics
 539 given various PINN model setups, our three physical features
 540 of interest, and whether the data has been differenced or not.
 541 Beyond providing an accurate forecast, we are primarily inter-
 542 ested in the regularization ability of the PINN's specialized loss
 543 function. As such, we begin by considering which values of
 544 λ yield the lowest error metrics. Then, the general forecasting
 545 ability of our highest performing models will be considered
 546 for further context. Finally, we will examine the buoy accuracy
 547 given its geographical region to consider where our method may
 548 struggle to provide high-quality outputs. In the Appendix, we
 549 supply Tables VII–XII to display the RMSE results gathered
 550 from our PINN models trained on various λ values. In the tables,
 551 each feature from horizons starting with 3 h (one period) and up
 552 to 24 h (eight periods) are given to see the evolution of error
 553 over time.

554 A. Selection of Best λ Values

555 We present the best value for λ given variations in our PINN
 556 models and the selected coupled feature. A series of figures
 557 display each λ value and corresponding error metrics per model
 558 and feature. We consider the original data best λ results for
 559 the GRU model in Fig. 1, the LSTM model in Fig. 2, and the
 560 Transformer model in Fig. 3. The λ -based ratio regularization
 561 successfully managed to reduce the MAE and RMSE of 24 h
 562 forecasts when compared with $\lambda = 1.0$ (no regularization). For
 563 the GRU and LSTM figures, each evaluated feature displays at
 564 least one value for λ , which yielded more performant metrics.
 565 Using the Transformer model, the PINN-style regularization
 566 yields explicitly worse forecasts for SST and Gust, but air
 567 pressure has a reduced error when $\lambda = 0.9$. In this sense, each
 568 model has displayed the property of MAE and RMSE reduction
 569 for at least one feature, using the regularization technique. The
 570 reason that the Transformer model performs well in the $\lambda = 1.0$

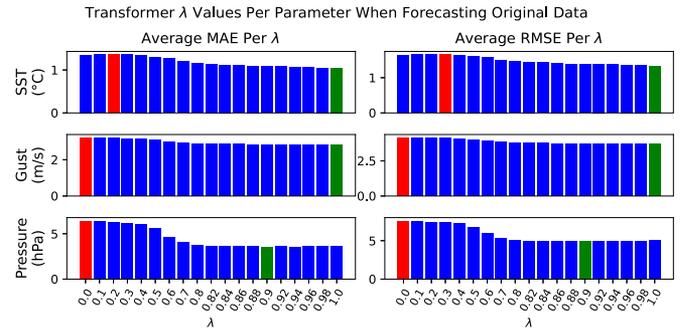


Fig. 3. MAE and RMSE for Transformer forecasts from $\lambda = 0.0$ to $\lambda = 1.0$ (no regularization). The lowest scoring λ value is displayed in green while the highest is red. Forecasts are given as the original values.

case is because the architecture is sufficiently complex enough
 to generalize the observations when trained using large amounts
 of data. However, the results of the air pressure forecasts imply
 some features benefit from the coupled loss function regardless
 of model complexity. The LSTM and GRU models are less com-
 plex and achieve worse test results overall, so the regularization
 has a larger effect on error reduction. For this reason, there exists
 a best performing model when $\lambda < 1.0$ in all features.

We highlight that the best λ values are unique for each
 experiment. This is true when comparing the separate features
 in the same model and when comparing the same feature from
 each model. For example, the best λ values found in the GRU
 features are 0.9, 0.84, and 0.96, for SST, gust strength, and air
 pressure, respectively. When comparing by model, the best λ
 for SST is largely separated at 0.9, 0.68, and 1.0 for GRU, LSTM,
 and Transformer models, respectively. The uniqueness of each
 λ selection is problematic in situations where the best λ value
 significantly differs between features. Each feature is coupled
 using the same λ value, although an optimal choice for one
 feature may not be optimal for all features. A multiple λ setup
 could allow more flexibility toward this problem.

In observing the change between λ values and their error
 metrics, we see some trends in each feature. The SST feature in
 GRU and LSTM models is inconsistent with many local minima
 observed. The gust strength feature displays error that is mostly
 consistent regardless of the selection of λ . However, there is

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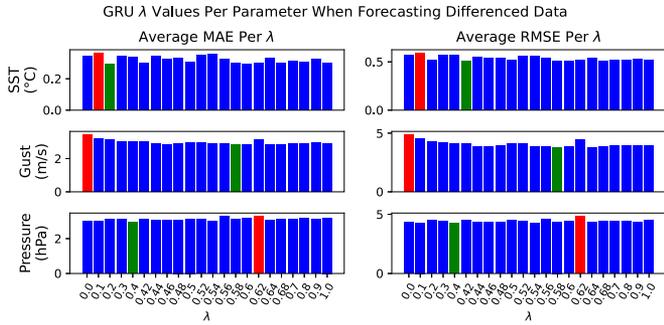


Fig. 4. MAE and RMSE for GRU forecasts from $\lambda = 0.0$ to $\lambda = 1.0$ (no regularization). The lowest scoring λ value is displayed in green while the highest is red. Forecasts are given as first-order differenced values.

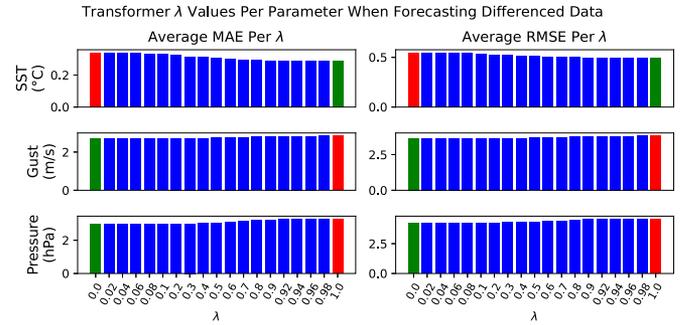


Fig. 6. MAE and RMSE for Transformer forecasts from $\lambda = 0.0$ to $\lambda = 1.0$ (no regularization). The lowest scoring λ value is displayed in green while the highest is red. Forecasts are given as first-order differenced values.

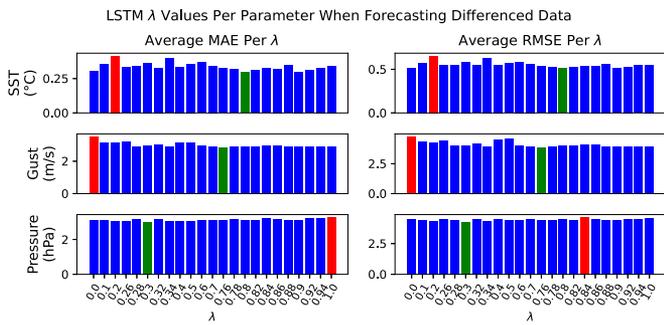


Fig. 5. MAE and RMSE for LSTM forecasts from $\lambda = 0.0$ to $\lambda = 1.0$ (no regularization). The lowest scoring λ value is displayed in green while the highest is red. Forecasts are given as first-order differenced values.

597 a noticeable decrease in error as λ approaches the discovered
 598 minimal value. The most obvious trend that occurs in all PINN
 599 models is the sharp decrease in error of the air pressure feature as
 600 λ increases. This is the sole case where a regularized Transformer
 601 model outperforms the $\lambda = 1.0$ case. This is likely caused by
 602 misalignment in the ERA5 model when compared with the
 603 ground truth. Extremely divergent outliers in ERA5 mean that
 604 training the surrogate model using numerical model data is a
 605 poor choice compared with the observations. So, error decreases
 606 when $\lambda > 0.5$ and the PINN produces forecasts more aligned
 607 with the observations. Still, the ERA5 data are well-fitted outside
 608 of outlier conditions, so $\lambda < 1.0$ promotes a regularizing effect
 609 on the model. This is an example of how our methodology can
 610 combine multiple data sources to improve results when each has
 611 their own biases.

612 Comparing the experimental results of the original data
 613 scheme to the results of the differenced data scheme shows
 614 varying results. We present the differenced data best λ results
 615 for the GRU model in Fig. 4, the LSTM model in Fig. 5, and the
 616 Transformer model in Fig. 6. The λ -based ratio regularization
 617 scheme reduces MAE and RMSE in all but one case. As before,
 618 the Transformer yields strictly better results when $\lambda = 1.0$ for
 619 SST. However, the ERA5 features show strictly best results when
 620 $\lambda = 0.0$, achieving lowest scores when the model is only trained
 621 on numerical data. Considering the GRU and LSTM figures,

each feature displays a minimizing λ that yields lower error
 metrics than the $\lambda = 1.0$ case. The best λ values found overall are
 typically closer to $\lambda = 0.0$. This is the exact opposite behavior
 when compared with the original results, and the trend is most
 obvious when considering the air pressure feature. Lower values
 of λ yield more performant results, although the absolute differ-
 ence in error is small. Most importantly, each model has shown
 error reduction for at least two features using the regularization
 technique.

The λ values for SST are chaotic, such as before, and the best
 value varies greatly per model. Conversely, the error metrics are
 much lower overall due to the differenced data representation.
 The behavior of λ regarding the gust strength feature is similar
 to the original data figures for the GRU and LSTM models. In
 all, the selection of a wider variety of lower λ values suggests
 that the rate of change in both datasets are alike. The numerical
 models also have less interpolated data, which promotes more
 stable training. Once again, we find that most results display
 best λ values, which are different between features and models.
 The one outlier comes from the Transformer model, where SST
 maintains a best result at $\lambda = 1.0$. Wind gust strength and air
 pressure both display similar values of λ between the GRU and
 LSTM models, but the SST varies drastically between each. This
 discussion underpins the idea that both the feature, the model,
 and the data representation influence the selection of best λ .

In this section we considered how the selection of the best λ
 differs as the parameters of our experiments change. The Trans-
 former model received the least benefit from $\lambda < 1.0$ overall.
 For the Transformer, the SST feature never benefits from the
 coupled loss, air pressure is always improved, and gust speed
 depends on whether the data are differenced or not. Both other
 models benefit at least somewhat from the regularization in
 all cases. We learned the benefit of the regularization and the
 corresponding selection of best λ are tied to the complexity
 of the model, where models with fewer weights benefit more
 when using this methodology. Another observation is that values
 approaching 0.0 for λ tend to yield worse results unless we are
 considering the differenced data representation. This is due to
 the way each model is trained to forecast the change between
 time steps. When taking a first-order difference of the data, a

TABLE IV
ORIGINAL VALUE FORECAST % CHANGE IN RMSE WHEN COMPARING THE
BEST FOUND λ AGAINST $\lambda = 1.0$ (NO REGULARIZATION) AND THE
NUMERICAL MODEL (HYCOM/ERA5)

Model	Best λ	Feature	$\lambda = 1.00$	Numerical Model
GRU	0.90	SST ($^{\circ}$ C)	-18.44%	+141.47%
	0.96	Pressure (hPa)	-14.83%	-3.08%
	0.84	Gust (m/s)	-3.98%	+33.11%
LSTM	0.68	SST ($^{\circ}$ C)	-15.42%	+145.45%
	0.82	Pressure (hPa)	-4.48%	-0.78%
	0.72	Gust (m/s)	-7.62%	+37.25%
Transformer	1.00	SST ($^{\circ}$ C)	0.0%	+102.02%
	0.90	Pressure (hPa)	-3.06%	-7.58%
	1.00	Gust (m/s)	0.0%	+26.44%

TABLE V
DIFFERENCED VALUE FORECAST % CHANGE IN RMSE WHEN COMPARING
THE BEST FOUND λ AGAINST $\lambda = 1.0$ (NO REGULARIZATION) AND THE
NUMERICAL MODEL (HYCOM/ERA5)

Model	Best λ	Feature	$\lambda = 1.00$	Numerical Model
GRU	0.42	SST ($^{\circ}$ C)	-1.64%	-23.43%
	0.40	Pressure (hPa)	-6.30%	-20.10%
	0.58	Gust (m/s)	-2.89%	+29.73%
LSTM	0.80	SST ($^{\circ}$ C)	-7.45%	-23.19%
	0.30	Pressure (hPa)	-7.18%	-19.66%
	0.76	Gust (m/s)	-2.18%	+31.46%
Transformer	1.00	SST ($^{\circ}$ C)	0.0%	-25.63%
	0.00	Pressure (hPa)	-7.93%	-20.72%
	0.00	Gust (m/s)	-4.80%	+24.45%

662 larger number of interpolated buoy observation values produces
663 an uninformative training environment for differenced data. The
664 numerical models, have fewer interpolated values and more
665 accurately reflect change from one time to another. Therefore,
666 PINNs which act more like the numerical model are more
667 performant in this case. Finally, by examining the way the best λ
668 changes in each experiment, we find that the feature, the model,
669 and the data representation all influence the selection of best λ .
670 Otherwise, the best λ selections would be more homogeneous
671 overall.

672 B. General Forecast Accuracy

673 By examining the general forecast accuracy of our models,
674 we gain additional insights into the coupled loss technique used
675 and the stability of our PINN models. To begin, we consider the
676 measured RMSE for the best found λ per feature. We compare
677 this error to those derived from the $\lambda = 1.0$ case and from
678 the numerical models for additional context. To facilitate this
679 comparison, we introduce Tables IV for the original value fore-
680 casts and V for the differenced value forecasts. In these tables,
681 we compare the percent change in RMSE between the best λ
682 value and $\lambda = 1.0$ in the fourth column. In the final column,

683 we compare the best λ value to the numerical models. These
684 values are calculated using the RMSE as found in the eight-step
685 forecast from the Appendix Tables VII–XII. Negative values
686 indicate a reduced error when comparing the best λ value to
687 the $\lambda = 1.0$ case or the numerical models. Positive values show
688 when the best λ results are worse than the compared source of
689 error. When the percentage is zero, the best value of λ for that
690 experiment was $\lambda = 1.0$.

691 Examining the original value forecast results in Table IV
692 tabulates that this method is rarely more performant than the
693 numerical models. The feature SST is worse than the numerical
694 model by at least 100%, which implies the HYCOM model is
695 well-calibrated to local conditions. When comparing the lower
696 resolution ERA5 model, air pressure and gust strength are less
697 aligned with the recorded observations. As a result, the feature
698 gust speed is up to 37% less accurate when using the PINN
699 models and results are more accurate using all architectures
700 for air pressure. This is encouraging and suggests that our
701 surrogate modeling technique can produce permissible forecasts
702 depending on the feature. The comparison of the best surrogate
703 model to the non-regularized surrogate when $\lambda = 1.0$ is more
704 favorable. From the Table, we show that there is a percent
705 decrease in error for most cases. The GRU and LSTM models are
706 more accurate when compared with the nonregularized versions.
707 The air pressure results show that the surrogate outperforms the
708 numerical model only after finding the best λ value. That is, we
709 only outperform the numerical model due to the coupled loss
710 function. The Transformer models showed improved forecasts
711 for air pressure alone. This indicates that a large network with
712 many trainable parameters can still benefit from our technique,
713 but the reduction in error will be less, if there is any at all.

714 Continuing, we consider the percent change in RMSE
715 when experimenting with the differenced data representation in
716 Table V. Overall, when comparing the PINN models to the
717 numeric model, we see improvement when using this data
718 representation. The only comparison, which is still worse than
719 the numerical models is when forecasting the gust speed feature,
720 although the percentage of error is slightly decreased. Almost
721 all the features show decrease in error when comparing the best
722 λ to the model trained when $\lambda = 1.0$. The spread of the decrease
723 in error is lesser than when forecasting the original data, with the
724 highest at about 8% and the lowest at 1.6%. There is no situation
725 for this data where the best λ directly causes improvement over
726 the numerical model, but we find an increased performance
727 gap between the deep learning and numerical models in most
728 cases.

729 We also consider the stability of the forecasts, given a single
730 example buoy. In Figs. 7 and 8, we show how the error of our
731 PINNs evolves over the forecast period of 24 h given chaotic
732 features, model architectures, and data representations. These
733 figures capture a subset of ten forecast periods, from time
734 steps 40 to 120, for a single buoy. The ground truth values are
735 reinitialized into the model every eighth time step, hence the ten
736 forecast periods. To select the λ value to represent in the figures,
737 we use the best λ value found for SST. When SST does not
738 have a best $\lambda < 1.0$, then the best value for gust strength or air
739 pressure was chosen. This highlights the limiting factor of our

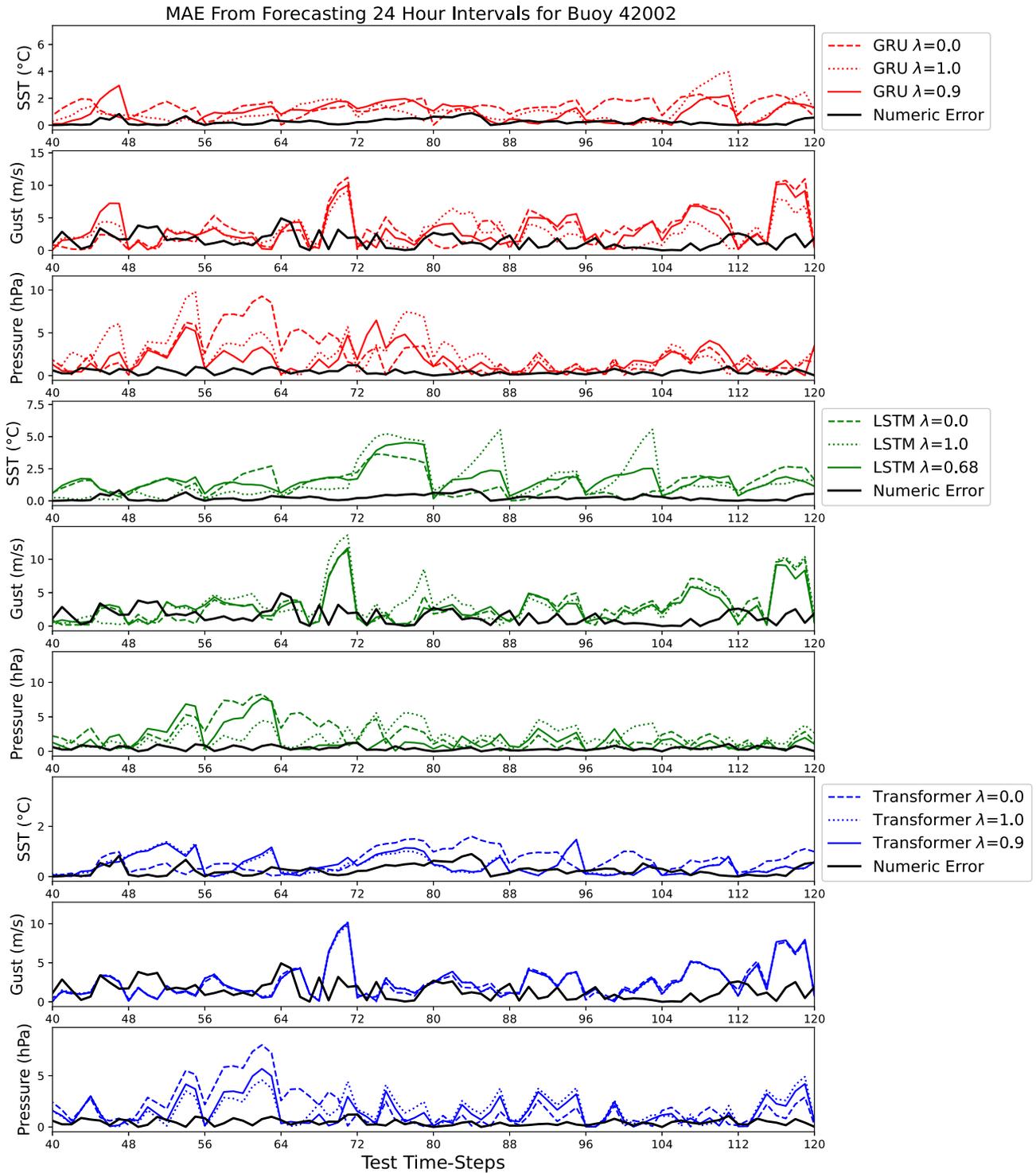


Fig. 7. Numerical and surrogate model MAE for each feature over ten 24-h forecast periods is displayed. We include each PINN with $\lambda = 0.0$, $\lambda = 1.0$ (no regularization), and the best found λ . The PINNs are reinitialized with new starting values every eighth period.

740 technique in its current form, as it cannot utilize multiple values
 741 for λ . Future explorations into this technique might consider a
 742 multiple λ setup for more flexibility.

743 When examining the original data forecast results for buoy
 744 42 002 in Fig. 7, it is expected for error to increase over the
 745 period. Ideally, the error of the best found λ will increase more

746 slowly than when $\lambda = 0.0$ or $\lambda = 1.0$, for each feature. From
 747 this figure, we can observe that error increases until the model is
 748 realigned with fresh initial values. We see that the forecasts are
 749 often worse than the numerical model. They are typically most
 750 performant around time steps one or two, when the initial values
 751 are still relatively recent. Comparing models and features shows

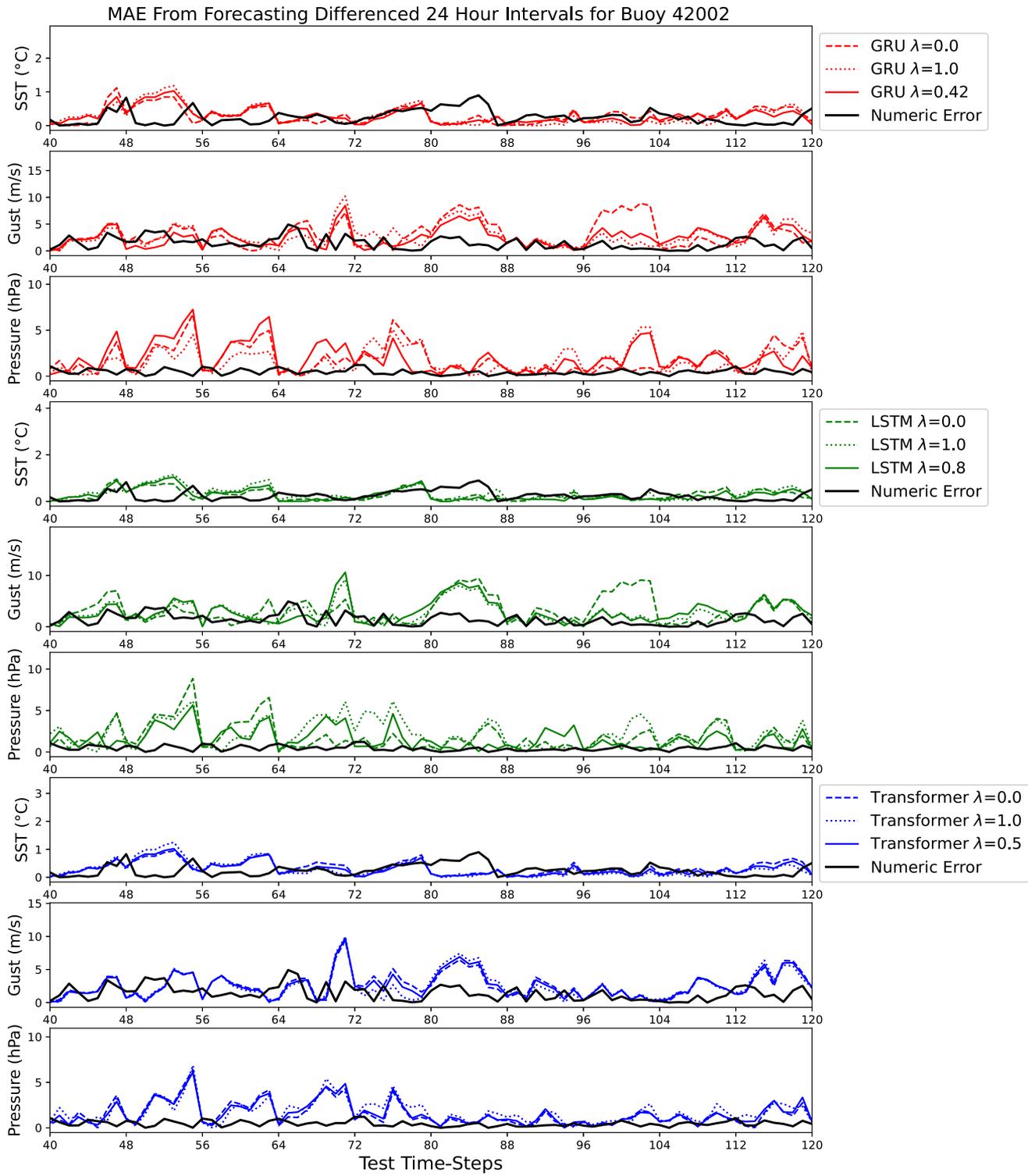


Fig. 8. Numerical and surrogate model MAE for each feature over ten 24-h forecast periods is displayed. Differenced value forecasts have been transformed back to the original scale before finding the error. We include each PINN with $\lambda = 0.0$, $\lambda = 1.0$ (no regularization), and the best found λ . The PINNs are reinitialized with new starting values every eighth period.

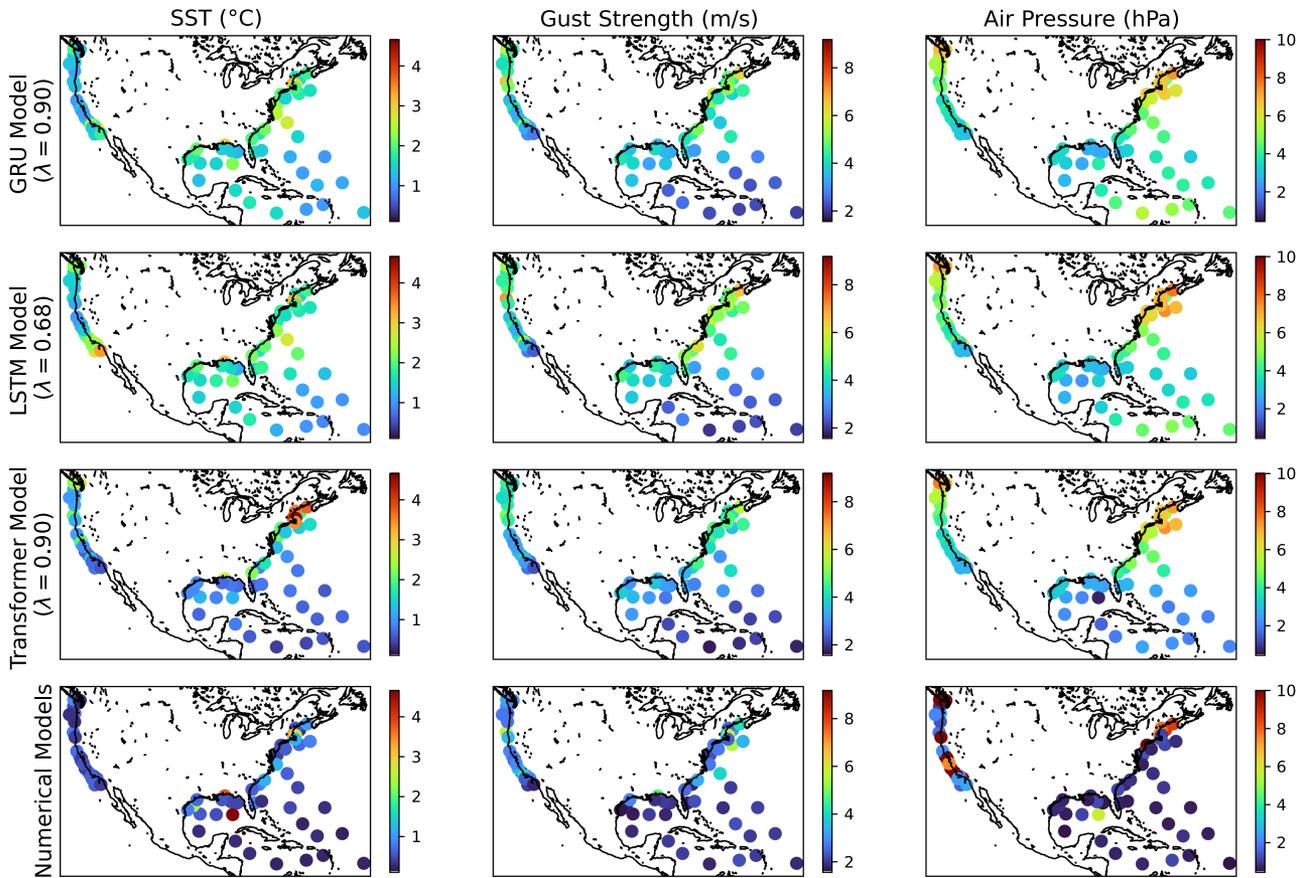


Fig. 9. Analyzed original features (columns) compared to the generating model (rows) by RMSE given at the geographical buoy location. Error is capped for SST and air pressure for visualization purposes. Color maps are normalized by each feature for comparative evaluation.

752 a wide variety of behaviors. The most similar forecasts are found
 753 when considering the Transformer, when each of the PINN
 754 models performs almost identically. The GRU models tend to
 755 disagree the most between each of the specific experiments,
 756 which makes sense considering it achieves the highest reduction
 757 in forecast error overall. PINNs are traditionally used to reduce
 758 numerical instability, and this behavior can be seen when fore-
 759 casting air pressure using the GRU model. Between time steps
 760 56 and 64, the best-selected λ shows significantly reduced error
 761 when comparing to the $\lambda = 1.0$ case. The same temporal region
 762 in the Transformer forecast displays the opposite behavior where
 763 the nonregularized model performs better than any regularized
 764 version. This is due to the complexity of the Transformer-
 765 based architecture, which causes the model to generalize un-
 766 derlying behaviors more effectively than the GRU or LSTM
 767 architectures.

768 Finally, we compare the differenced value forecast MAE
 769 scores for buoy 42 002 from the Fig. 8. In the case of the
 770 Transformer model, we show $\lambda = 0.5$ because each feature's
 771 best λ lies on the extreme end of either $\lambda = 0.0$ or $\lambda = 1.0$.
 772 The main benefit of using the differenced data representation is
 773 displayed by the reduction in overall error across all models. The
 774 figure demonstrates how the λ forces the PINN to behave more
 775 like one data source or the other, evidenced by the fact that the

776 MAE found tends to be bound by the other error sources. Overall,
 777 error increases more slowly in regions where the forecasted
 778 feature remains highly stable over time. Once again, we see that
 779 refreshing the initial values reduces error significantly, which is
 780 the expected behavior. The error spread between the PINN is
 781 much more similar in this case because the models rely more
 782 on autocorrelation between forecast periods. Error reduction
 783 is significant enough to suggest the regularized models make
 784 more informed forecasts on average. It is significant to note
 785 that individual plots of forecasts from the best λ model may be
 786 less accurate than other setups in specific instances, but error is
 787 reduced overall when considering all buoys.

788 In this section, we analyzed the forecasting ability of our
 789 models by considering percent reduction in errors and the fore-
 790 cast of a single buoy via different experimental permutations.
 791 The selection of λ and total amount of error reduction was
 792 shown to depend on the model, the features examined, and the
 793 data representation used. When compared with models where
 794 $\lambda = 1.0$, percentage reductions in error were as low as 1.6%
 795 and as high as 18.4%. When using the Transformer model, the
 796 feature SST never showed improvement over the $\lambda = 1.0$ case.
 797 The surrogate models always outperform the numerical model
 798 for the air pressure feature and outperform in SST forecasting
 799 depending on the data representation. We never outperform the

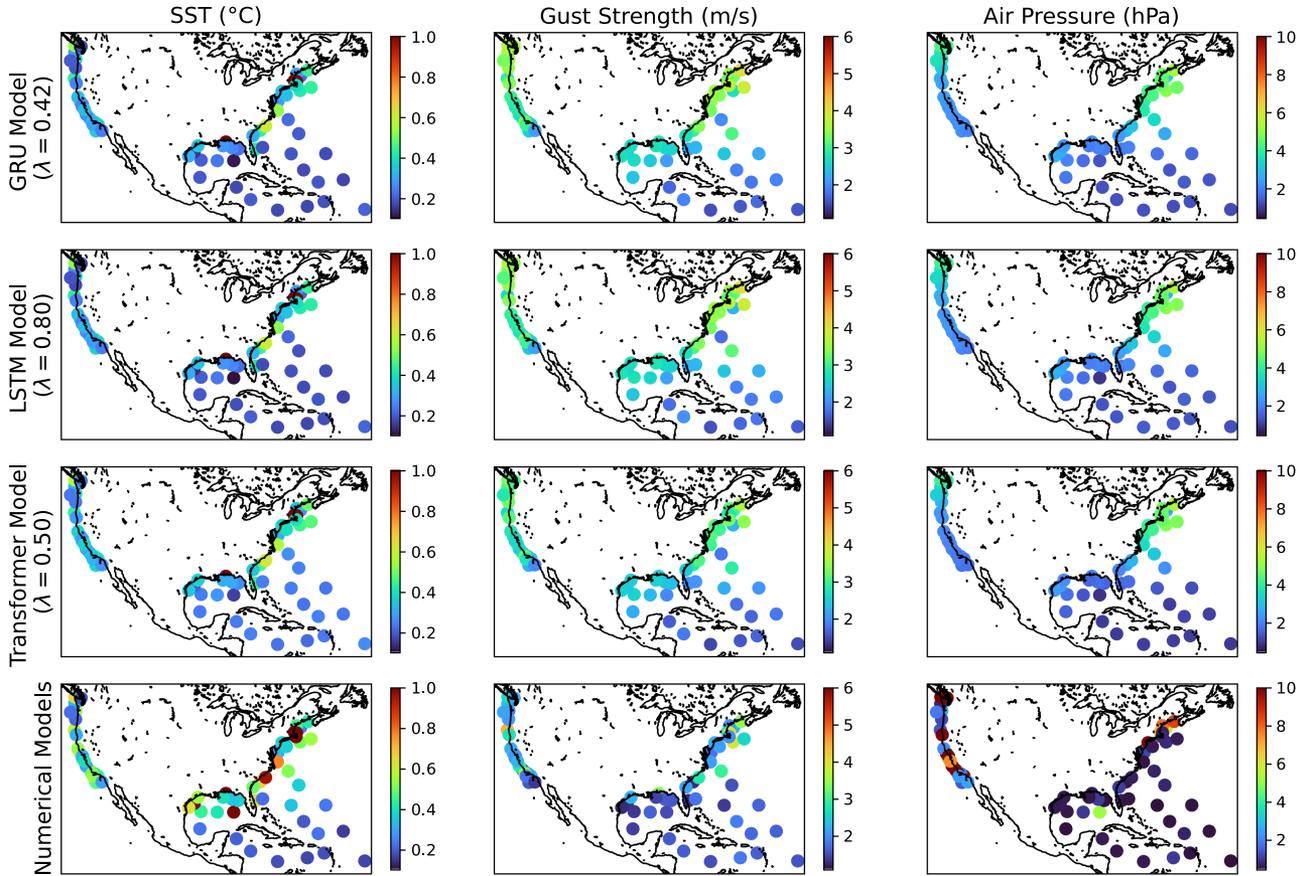


Fig. 10. Analyzed differenced features (columns) compared to the generating model (rows) by RMSE given at the geographical buoy location. Error is capped for SST and air pressure for visualization purposes. Color maps are normalized by each feature for comparative evaluation.

800 numerical model when forecasting gust strength. In the case of
 801 feature air pressure, the error reduction from selecting λ through
 802 a grid search allows the surrogate PINN model to out-perform the
 803 numerical model. It is important to restate that the interpolated
 804 values in the ground truth provide some bias in the test by
 805 penalizing the numerical models when comparing to those
 806 interpolated values. In addition, inference based on differenced
 807 inputs produces more stable estimates of local conditions, i.e.,
 808 the observations. Our surrogate models benefit from both points,
 809 which explains the general improvement when compared with
 810 the numerical model. More importantly, selecting the best
 811 regularization parameter, λ , yields models that achieve higher
 812 accuracy, and this is consistent across both data representations.
 813 We showed how the error in forecasts are reduced on average
 814 by training the surrogate model using the selected λ value. This
 815 revealed the way model selection and data representation affects
 816 the numerical stability over the forecast period. The differenced
 817 data representation simplifies the problem for the surrogate
 818 models, so the forecast stability remains similar between models
 819 and features. The opposite is true in the original data forecasts,
 820 which is more chaotic and showed disagreements. In all, the
 821 analysis of these results suggest that our model is relatively
 822 stable over 24 h periods, but error is often worse than the
 823 reanalysis models when they are well-fitted to the observation
 824 data.

C. Geographical Error Analysis

825
 826 Our final method for comparing the numerical models with
 827 our PINNs involves an analysis of buoy RMSE per their geo-
 828 graphical position. To this end, we provide two figures, which
 829 represent a grid of our models as rows with the forecasted
 830 feature as columns. Positional markers reference the latitude and
 831 longitude of each buoy, and there is overlap due to the number of
 832 buoys. The color bar represents the amount of RMSE calculated
 833 for a buoy and is normalized column-wise by the minimum and
 834 maximum error generated for the feature by each model. In Fig. 9,
 835 we show the results from the original data forecast and in Fig. 10,
 836 we show the results from the differenced dataset. One
 837 caveat to these figures is that we cap the error of the air pressure
 838 feature in both figures to a max value of 10. This is because
 839 the ERA5 has an extreme misalignment in outlier areas, which
 840 dominates the color interpolation. We cap the error derived from
 841 SST to a max value of one in the differenced Fig. 10 for the same
 842 reason.

843 The original values forecast results in Fig. 9 show there are
 844 some trends among the models. First, the best performing region
 845 for all features are the forecasts of buoys clustered around the
 846 Caribbean. The Gulf of Mexico region performs similarly but
 847 can be slightly less accurate depending on the experiment. The
 848 least performant regions tend to be along the North Atlantic
 849 East-Coast and various regions around the Pacific West-Coast.
 849

The numerical models are, on average, are extremely well fitted to real-world observations. Although, there are cases, possibly due to resolution constraints of grid data, where massive influxes of error are found. This misalignment shows the benefit of local condition forecasting. For example, the numerically modeled outliers for air pressure are along the West-Coast. These same regions perform well using our technique because we model the forecast based on local observed conditions. Geographic regions, which are poorly forecasted by a PINN model tend to cluster among similarly performing regions. We do not observe alternating high- and low-error regions, which would imply random forecasts. Instead, we very consistently see gradients of low- to high-error regions. This may be explained by considering that some regions may pose a modeling challenge due to geography, river runoff, human operations, lack of data, and so on.

Next, we analyze the difference valued forecast results in Fig. 10. The results are more homogeneous and more accurate across all models and features. Compared with the original forecast, similar geographical zones display relatively high errors, showing these are likely regions of high change. Each of the PINN models yields similar error scores, which suggests that they rely on low-change forecasts to accurately describe the true value. Therefore, the models produce more similar results and are more sensitive to chaotic regions. From the figure, we can pick out an instance of an outlier buoy in the center of the Caribbean region, when forecasting the SST parameter. There, error from HYCOM is high while the error from each PINN model is low. In this case, the numerical model represents real world conditions and error is calculated through interpolated initial values, causing inflated metrics. However, this is not the reason for all outliers. In the case of air pressure, most high-error regions are a case of misalignment in the numerical model.

By examining the individual buoy error, we learned which geographic regions are most difficult to model. We also revealed patterns in the similarities between our PINN experiments and the numerical models. The figures revealed that the numerical models have some regions with high error. The error is mainly found when there is misalignment in the numerical models. Some error was introduced through our interpolation scheme, such as the SST outlier in the Gulf of Mexico. Buoys, which received low accuracy forecasts tend to be surrounded by buoys with similar metrics, which implies they are within difficult-to-model geographical regions. Although the error for the differenced data representation is lower than when forecasting the original values, the buoys with the highest error come from similar regions. When comparing our sparse forecasting technique to a full-coverage model, our method is not constrained to a grid region, and any arbitrary point may be modeled. Therefore, error may be reduced when forecasting regions between vertices, without relying on interpolation techniques. The drawback of using this sparse forecasting technique is that greater spatial conditions cannot be deciphered by the observations alone. In this way, we tradeoff providing regional context to the PINN model for increased forecasting flexibility. The PINN architecture bases the forecast off current conditions alone and is independent of the buoy's geography.

V. CONCLUSION

We investigated the ability of the ocean flow model HYCOM and the climate model ERA5 to be used as regularization data for PINN-inspired deep learning models. A special formulation of the loss function yielded comprehensive models for forecasting any number of physical parameters in a sequence-to-sequence model. The techniques demonstrated how multiple ocean and climate features may be forecasted and combined using deep LSTM, GRU, and Transformer physics-informed networks. Our sparse feature forecasting approach yielded more flexible, generalized models, which are less constrained to predefined regions. In contrast to other PINN models, we train the models using observation data while regularizing with precomputed numerical models. The significance of this is that we do not need to implement the numerical formulation for use in our framework. In most cases, we improved the surrogate model performance by combining the observation data and numerical models. To assess the models, we set up experimental sparse sequential forecasting procedures for SST, air pressure, and gust strength as observed by free floating buoys. Two separate data representations were investigated, which included the original observed/modeled data and first order differenced versions of the data. Over these experiments, the hyperparameter λ was fine-tuned between 0.0 and 1.0 to find the best possible data ratio. We found that models, which have a less complex architecture improved the most from the inclusion of the numerical model regularization. This was shown explicitly by comparing the results of the least complex and most complex architectures of the GRU and Transformer models. The GRU and LSTM models showed improvements after tuning for λ in every case while the Transformer models showed improvement for fewer features. Further, the selection of λ significantly altered the behavior of the PINN models. As the λ value approaches 0.0, the trained model produced results more like the numerical models, while the opposite is true when λ approaches 1.0. Depending on the experiment, we saw improvements over the numerical model in forecast error. In favor of our method, the PINN forecasting of air pressure showed improvement over the numerical models when the best selection of λ was chosen. Overall, our method improved the numerical stability of the forecasts on average over the horizon period. In the case of the differenced data representation, we saw the stability of each PINN model was similar. Lower valued λ values were most performant in this case, which suggests the numerical model data was more informative overall. This is likely due to fewer interpolated values from the numerical models when compared with the buoy observations. The differenced data forecasts are the most accurate overall, but the amount of error reduction found when using this data representation was less. Exploring the error geographically showed us that modeling high-change areas of interest is difficult for both the numerical models and our PINNs. This methodology can be used to forecast observations between the vertices of grid-based numerical models. The tradeoff of the increased flexibility is the loss of context of spatial conditions beyond the immediate forecast region. Ongoing work on this methodology continues in several ways. Because the selection of λ changes on a feature-by-feature basis, we should investigate

962 an approach to allow an independent selection of λ values on a
 963 per-feature case. Using a grid search for selecting the best λ value
 964 is currently inefficient. Future improvements to our technique
 965 will revolve around fine-tuning the λ selection approach to
 966 reduce computational overhead of the models. Moreover, since
 967 we formulate new models that combine numerical models with
 968 observations, our framework leaves room to explore integration
 969 into a data assimilation scheme. The methodology should be
 970 expanded to combine multiple numerical models with relevant
 971 PDEs to see if similar improvements can be found when forecast-
 972 ing full-coverage models also. Different domain problems and
 973 experimental setups will yield further insight into this procedure
 974 for combining multiple sources of data when each has inherent
 975 limitations.

976 APPENDIX A

977 124 selected buoy observations from the NOAA archive for
 978 potential inclusion into train, validation, and test datasets. The
 979 numbers selected into each set are displayed in Table VI.

980 51001, 41002, 41004, 41008, 41009, 41010, 41013, 41025,
 981 41040, 41041, 41043, 41044, 41046, 41047, 41048, 41049,
 982 42001, 42002, 42003, 42012, 42019, 42020, 42035, 42036,
 983 42039, 42040, 42055, 42056, 42057, 42058, 42059, 42060,
 984 44005, 44007, 44008, 44009, 44011, 44013, 44014, 44017,
 985 44018, 44020, 44025, 44027, 44065, 44066, 45001, 45002,
 986 45003, 45004, 45005, 55039, 45006, 45007, 45008, 45012,
 987 46001, 46002, 46005, 46006, 46011, 46012, 46013, 46014,
 988 46015, 46022, 46025, 46026, 46027, 46028, 46029, 46035,
 989 46041, 46042, 46047, 46050, 46053, 46054, 46059, 46060,
 990 46061, 46066, 46069, 46070, 46071, 46072, 46073, 46075,
 991 46076, 46077, 46078, 46080, 46081, 46082, 46083, 46084,
 992 46085, 46086, 46087, 46088, 46089, 51000, 51001, 51002,
 993 51003, 51004, 51101, 46221, 46214, 46211, 46224, 46215,
 994 46222, 46213, 46235. 46239, 46240, 46243, 46244, 46232,
 995 44095, 44100, 42099, and 44024.

TABLE VI
 NUMBER OF BUOYS DISTRIBUTED INTO EACH DATASET

Subset Contributions by Buoy	Total Number
Total Buoys	124
Train Only	3
Val Only	0
Test Only	1
Train and Test Only	2
Val and Test Only	1
Train/Test/Val Included	86
Not Included At All	31

There are 127 buoys sorted in total.

TABLE VII
 GRU ORIGINAL FORECASTS PER $\lambda \in [0, 1]$ RMSE RESULTS OVER EIGHT
 FORECAST PERIODS (24 H)

Feature	λ	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000
SST (°C)	0.00	1.117	1.327	1.507	1.663	1.800	1.923	2.035	2.138
	0.10	1.029	1.216	1.370	1.503	1.619	1.724	1.818	1.907
	0.20	1.006	1.178	1.324	1.452	1.567	1.670	1.764	1.850
	0.30	0.986	1.195	1.372	1.529	1.670	1.798	1.918	2.029
	0.40	0.978	1.198	1.387	1.552	1.697	1.826	1.941	2.045
	0.50	0.855	1.038	1.194	1.329	1.449	1.558	1.660	1.757
	0.60	0.828	1.030	1.197	1.342	1.471	1.587	1.691	1.785
	0.70	0.882	1.143	1.370	1.574	1.761	1.932	2.091	2.238
	0.80	0.851	1.067	1.239	1.384	1.508	1.618	1.714	1.801
	0.90	0.781	0.977	1.134	1.262	1.369	1.460	1.539	1.607
1.00	0.887	1.133	1.332	1.497	1.640	1.763	1.872	1.970	
Pressure (hPa)	0.00	6.223	6.663	7.011	7.306	7.569	7.805	8.016	8.202
	0.10	6.240	6.702	7.054	7.344	7.593	7.812	8.004	8.175
	0.20	6.393	7.038	7.536	7.947	8.297	8.599	8.858	9.081
	0.30	6.072	6.643	7.077	7.432	7.736	7.999	8.224	8.419
	0.40	5.746	6.424	6.972	7.437	7.837	8.179	8.467	8.713
	0.50	4.446	5.194	5.753	6.202	6.579	6.898	7.169	7.402
	0.60	2.896	3.632	4.252	4.798	5.285	5.711	6.079	6.401
	0.70	2.343	2.968	3.507	4.013	4.508	4.971	5.383	5.754
	0.80	2.302	2.882	3.378	3.831	4.273	4.692	5.073	5.420
	0.96	2.072	2.657	3.148	3.598	4.037	4.447	4.817	5.154
1.00	2.119	2.832	3.452	4.034	4.600	5.136	5.617	6.051	
Gust (m/s)	0.00	3.044	3.399	3.709	3.975	4.205	4.405	4.580	4.738
	0.10	2.917	3.256	3.554	3.811	4.029	4.212	4.366	4.501
	0.20	2.957	3.312	3.616	3.873	4.090	4.271	4.425	4.560
	0.30	2.809	3.124	3.388	3.606	3.787	3.938	4.065	4.176
	0.40	2.789	3.138	3.438	3.691	3.903	4.077	4.223	4.348
	0.50	2.683	3.076	3.404	3.678	3.906	4.094	4.251	4.387
	0.60	2.538	2.963	3.285	3.541	3.747	3.916	4.059	4.182
	0.70	2.412	2.806	3.107	3.347	3.541	3.700	3.833	3.947
	0.84	2.396	2.782	3.077	3.309	3.497	3.650	3.781	3.894
	0.90	2.415	2.841	3.167	3.429	3.640	3.813	3.958	4.081
1.00	2.378	2.778	3.102	3.368	3.587	3.768	3.923	4.055	

TABLE VIII
LSTM ORIGINAL FORECASTS PER $\lambda \in [0, 1]$ RMSE RESULTS OVER EIGHT FORECAST PERIODS (24 H)

Feature	λ	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000
SST (°C)	0.00	1.089	1.282	1.459	1.616	1.754	1.875	1.983	2.080
	0.10	1.031	1.237	1.418	1.583	1.733	1.867	1.989	2.102
	0.20	1.052	1.241	1.408	1.551	1.672	1.776	1.865	1.943
	0.30	1.120	1.344	1.533	1.694	1.833	1.954	2.060	2.153
	0.40	0.900	1.101	1.272	1.418	1.545	1.658	1.760	1.856
	0.50	0.813	1.005	1.165	1.298	1.415	1.519	1.613	1.700
	0.60	0.788	1.015	1.201	1.360	1.497	1.617	1.723	1.817
	0.68	0.756	0.962	1.127	1.263	1.377	1.475	1.560	1.633
	0.80	0.773	1.001	1.186	1.342	1.478	1.597	1.705	1.805
	0.90	0.798	1.033	1.226	1.388	1.527	1.648	1.753	1.848
1.00	0.850	1.097	1.296	1.462	1.603	1.727	1.835	1.931	
Pressure (hPa)	0.00	6.706	7.270	7.691	8.020	8.288	8.510	8.698	8.858
	0.10	6.371	6.854	7.220	7.517	7.770	7.987	8.176	8.343
	0.20	6.493	7.150	7.666	8.079	8.418	8.700	8.938	9.140
	0.30	6.334	7.070	7.646	8.117	8.517	8.862	9.162	9.424
	0.40	5.788	6.556	7.155	7.653	8.083	8.460	8.791	9.084
	0.50	4.557	5.382	6.017	6.532	6.960	7.316	7.613	7.865
	0.60	2.675	3.410	4.037	4.596	5.101	5.546	5.932	6.269
	0.70	2.472	3.119	3.670	4.163	4.617	5.026	5.385	5.703
	0.82	2.241	2.832	3.319	3.762	4.190	4.594	4.954	5.276
	0.90	2.215	2.817	3.315	3.767	4.205	4.616	4.983	5.310
1.00	2.038	2.656	3.186	3.682	4.183	4.672	5.120	5.524	
Gust (m/s)	0.00	2.944	3.240	3.499	3.717	3.904	4.062	4.197	4.315
	0.10	2.991	3.323	3.602	3.831	4.021	4.179	4.310	4.422
	0.20	2.931	3.260	3.536	3.767	3.962	4.128	4.273	4.402
	0.30	2.836	3.169	3.455	3.697	3.902	4.075	4.225	4.355
	0.40	2.768	3.107	3.399	3.647	3.857	4.034	4.185	4.319
	0.50	2.666	3.018	3.314	3.557	3.756	3.919	4.054	4.168
	0.60	2.535	2.976	3.315	3.584	3.805	3.986	4.136	4.264
	0.72	2.440	2.840	3.148	3.397	3.598	3.762	3.898	4.015
	0.80	2.458	2.914	3.270	3.559	3.798	3.998	4.169	4.318
	0.90	2.413	2.842	3.191	3.478	3.714	3.911	4.077	4.217
1.00	2.386	2.834	3.202	3.514	3.776	3.998	4.186	4.346	

TABLE IX
TRANSFORMER ORIGINAL FORECASTS PER $\lambda \in [0, 1]$ RMSE RESULTS OVER EIGHT FORECAST PERIODS (24 H)

Feature	λ	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000
SST (°C)	0.00	0.918	1.027	1.134	1.240	1.345	1.448	1.547	1.644
	0.10	0.893	1.008	1.121	1.233	1.344	1.451	1.556	1.658
	0.20	0.863	0.985	1.102	1.219	1.334	1.446	1.554	1.659
	0.30	0.829	0.957	1.081	1.204	1.324	1.441	1.555	1.664
	0.40	0.784	0.916	1.044	1.171	1.296	1.418	1.537	1.652
	0.50	0.722	0.852	0.982	1.114	1.244	1.372	1.498	1.620
	0.60	0.668	0.808	0.943	1.078	1.211	1.340	1.464	1.583
	0.70	0.615	0.753	0.885	1.017	1.147	1.273	1.395	1.512
	0.80	0.587	0.724	0.853	0.981	1.107	1.230	1.348	1.463
	0.90	0.572	0.699	0.818	0.936	1.052	1.166	1.276	1.383
1.00	0.568	0.691	0.805	0.918	1.030	1.138	1.243	1.344	
Pressure (hPa)	0.00	6.204	6.447	6.663	6.872	7.076	7.269	7.448	7.613
	0.10	6.060	6.334	6.565	6.783	6.995	7.195	7.379	7.548
	0.20	5.878	6.192	6.441	6.671	6.893	7.100	7.290	7.464
	0.30	5.628	6.005	6.284	6.533	6.767	6.984	7.180	7.358
	0.40	5.240	5.735	6.070	6.353	6.611	6.845	7.053	7.242
	0.50	4.050	4.817	5.326	5.718	6.051	6.338	6.587	6.807
	0.60	2.684	3.421	3.995	4.490	4.934	5.325	5.666	5.966
	0.70	2.231	2.877	3.397	3.865	4.306	4.709	5.063	5.379
	0.80	2.003	2.592	3.069	3.512	3.947	4.354	4.716	5.040
	0.90	1.885	2.446	2.904	3.336	3.775	4.196	4.574	4.914
1.00	1.824	2.391	2.867	3.324	3.799	4.263	4.685	5.070	
Gust (m/s)	0.00	2.901	3.145	3.367	3.567	3.745	3.902	4.042	4.169
	0.10	2.844	3.102	3.332	3.537	3.721	3.882	4.027	4.158
	0.20	2.778	3.052	3.292	3.505	3.696	3.864	4.014	4.150
	0.30	2.699	2.992	3.240	3.459	3.653	3.823	3.974	4.111
	0.40	2.595	2.916	3.177	3.403	3.601	3.774	3.927	4.064
	0.50	2.466	2.827	3.107	3.342	3.545	3.720	3.873	4.009
	0.60	2.321	2.707	3.005	3.251	3.459	3.637	3.791	3.927
	0.70	2.229	2.615	2.917	3.165	3.374	3.551	3.704	3.840
	0.80	2.175	2.555	2.855	3.103	3.311	3.488	3.641	3.777
	0.90	2.150	2.522	2.818	3.064	3.270	3.445	3.596	3.730
1.00	2.138	2.505	2.798	3.042	3.246	3.418	3.567	3.698	

TABLE X
GRU DIFFERENCED FORECASTS PER $\lambda \in [0, 1]$ RMSE RESULTS OVER EIGHT FORECAST PERIODS (24 H)

Feature	λ	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000
SST (°C)	0.00	0.220	0.339	0.400	0.451	0.493	0.525	0.549	0.569
	0.10	0.221	0.342	0.410	0.467	0.513	0.549	0.575	0.595
	0.20	0.211	0.330	0.384	0.424	0.455	0.479	0.498	0.515
	0.30	0.225	0.349	0.414	0.463	0.501	0.529	0.551	0.569
	0.40	0.218	0.341	0.402	0.448	0.486	0.517	0.543	0.566
	0.50	0.211	0.330	0.382	0.421	0.453	0.480	0.502	0.523
	0.60	0.219	0.339	0.392	0.430	0.458	0.480	0.496	0.510
	0.70	0.218	0.339	0.393	0.432	0.462	0.486	0.505	0.522
	0.80	0.216	0.336	0.389	0.426	0.455	0.479	0.499	0.517
	0.90	0.218	0.337	0.391	0.430	0.461	0.487	0.510	0.532
1.00	0.222	0.340	0.391	0.427	0.455	0.478	0.499	0.518	
Pressure (hPa)	0.00	1.044	1.498	2.003	2.516	3.028	3.504	3.923	4.304
	0.10	1.043	1.474	1.970	2.490	3.010	3.491	3.911	4.291
	0.20	1.059	1.496	1.994	2.514	3.052	3.571	4.049	4.496
	0.30	1.101	1.582	2.094	2.620	3.153	3.635	4.048	4.422
	0.40	1.085	1.536	2.021	2.517	3.016	3.474	3.878	4.249
	0.50	1.149	1.631	2.132	2.647	3.167	3.646	4.069	4.464
	0.58	1.151	1.697	2.214	2.705	3.183	3.622	4.015	4.377
	0.70	1.183	1.699	2.198	2.678	3.162	3.622	4.037	4.429
	0.80	1.224	1.801	2.316	2.796	3.262	3.695	4.078	4.433
	0.90	1.231	1.797	2.297	2.754	3.195	3.613	3.996	4.360
1.00	1.276	1.864	2.365	2.808	3.257	3.720	4.151	4.534	
Gust (m/s)	0.00	2.089	2.668	3.176	3.623	4.006	4.343	4.637	4.901
	0.10	2.122	2.645	3.095	3.481	3.803	4.073	4.298	4.496
	0.20	2.032	2.489	2.880	3.219	3.517	3.786	4.035	4.273
	0.30	2.052	2.511	2.903	3.244	3.535	3.786	4.005	4.201
	0.40	2.101	2.539	2.914	3.228	3.492	3.718	3.912	4.082
	0.50	2.120	2.532	2.883	3.188	3.454	3.687	3.895	4.081
	0.60	2.154	2.541	2.870	3.143	3.369	3.556	3.716	3.856
	0.70	2.222	2.579	2.891	3.158	3.385	3.578	3.747	3.897
	0.80	2.318	2.650	2.947	3.204	3.422	3.605	3.763	3.900
	0.90	2.434	2.759	3.048	3.297	3.508	3.682	3.832	3.962
1.00	2.446	2.756	3.032	3.269	3.470	3.639	3.782	3.908	

TABLE XI
LSTM DIFFERENCED FORECASTS PER $\lambda \in [0, 1]$ RMSE RESULTS OVER EIGHT FORECAST PERIODS (24 H)

Feature	λ	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000
SST (°C)	0.00	0.214	0.330	0.384	0.424	0.456	0.481	0.501	0.518
	0.10	0.222	0.341	0.405	0.454	0.493	0.525	0.551	0.574
	0.20	0.225	0.356	0.431	0.493	0.546	0.587	0.621	0.652
	0.30	0.220	0.344	0.409	0.458	0.498	0.530	0.557	0.581
	0.40	0.211	0.332	0.389	0.433	0.470	0.501	0.528	0.553
	0.50	0.216	0.336	0.392	0.435	0.474	0.509	0.541	0.570
	0.60	0.222	0.348	0.412	0.459	0.497	0.529	0.557	0.583
	0.70	0.218	0.341	0.397	0.439	0.474	0.505	0.533	0.559
	0.80	0.218	0.338	0.389	0.425	0.452	0.475	0.494	0.511
	0.90	0.215	0.336	0.388	0.424	0.452	0.476	0.495	0.512
1.00	0.230	0.352	0.408	0.448	0.480	0.507	0.531	0.552	
Pressure (hPa)	0.00	1.027	1.463	1.974	2.502	3.035	3.548	4.022	4.461
	0.10	1.065	1.526	2.058	2.612	3.152	3.634	4.038	4.400
	0.20	1.067	1.553	2.068	2.587	3.102	3.567	3.968	4.331
	0.30	1.104	1.579	2.087	2.592	3.085	3.525	3.912	4.272
	0.40	1.097	1.542	2.030	2.542	3.064	3.557	4.015	4.456
	0.50	1.160	1.655	2.147	2.629	3.117	3.576	3.993	4.386
	0.60	1.176	1.710	2.219	2.711	3.195	3.639	4.026	4.387
	0.70	1.180	1.706	2.199	2.666	3.144	3.628	4.088	4.518
	0.76	1.219	1.759	2.262	2.737	3.200	3.635	4.037	4.418
	0.90	1.246	1.795	2.271	2.708	3.159	3.620	4.045	4.434
1.00	1.320	1.913	2.415	2.865	3.322	3.788	4.215	4.603	
Gust (m/s)	0.00	2.059	2.590	3.056	3.469	3.834	4.161	4.463	4.758
	0.10	2.067	2.584	3.023	3.392	3.701	3.957	4.170	4.349
	0.20	2.073	2.561	2.982	3.338	3.633	3.887	4.112	4.313
	0.30	2.077	2.539	2.922	3.240	3.502	3.720	3.906	4.067
	0.40	2.106	2.557	2.968	3.344	3.683	3.993	4.280	4.536
	0.50	2.133	2.584	3.000	3.383	3.729	4.050	4.354	4.638
	0.60	2.197	2.580	2.911	3.198	3.443	3.653	3.840	4.009
	0.70	2.267	2.635	2.955	3.229	3.463	3.660	3.831	3.982
	0.80	2.369	2.703	3.003	3.267	3.492	3.683	3.850	3.998
	0.90	2.393	2.713	3.006	3.260	3.473	3.652	3.806	3.939
1.00	2.470	2.782	3.061	3.299	3.498	3.664	3.806	3.931	

TABLE XII
TRANSFORMER DIFFERENCED FORECASTS PER $\lambda \in [0, 1]$ RMSE RESULTS
OVER EIGHT FORECAST PERIODS (24 H)

Feature	λ	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000
SST (°C)	0.00	0.206	0.322	0.380	0.426	0.463	0.494	0.522	0.549
	0.10	0.205	0.321	0.378	0.421	0.456	0.486	0.512	0.539
	0.20	0.204	0.321	0.376	0.417	0.450	0.479	0.504	0.530
	0.30	0.204	0.320	0.374	0.413	0.445	0.472	0.497	0.522
	0.40	0.203	0.320	0.373	0.410	0.441	0.468	0.492	0.516
	0.50	0.202	0.320	0.371	0.408	0.438	0.464	0.488	0.511
	0.60	0.202	0.320	0.370	0.406	0.435	0.461	0.485	0.507
	0.70	0.201	0.319	0.370	0.404	0.433	0.458	0.482	0.504
	0.80	0.201	0.320	0.369	0.404	0.432	0.457	0.479	0.500
	0.90	0.201	0.320	0.369	0.403	0.431	0.456	0.477	0.497
1.00	0.201	0.320	0.370	0.404	0.431	0.455	0.476	0.495	
Pressure (hPa)	0.00	0.933	1.410	1.940	2.470	2.980	3.440	3.847	4.216
	0.10	0.950	1.438	1.966	2.494	3.005	3.463	3.865	4.231
	0.20	0.965	1.465	1.993	2.519	3.029	3.484	3.883	4.246
	0.30	0.984	1.501	2.031	2.553	3.060	3.511	3.904	4.265
	0.40	1.007	1.544	2.079	2.596	3.095	3.541	3.931	4.289
	0.50	1.032	1.594	2.134	2.646	3.136	3.575	3.964	4.320
	0.60	1.059	1.649	2.197	2.700	3.178	3.614	4.003	4.357
	0.70	1.091	1.712	2.269	2.761	3.227	3.662	4.054	4.404
	0.80	1.125	1.783	2.348	2.826	3.281	3.723	4.119	4.461
	0.90	1.160	1.855	2.426	2.889	3.335	3.787	4.184	4.517
1.00	1.197	1.933	2.511	2.958	3.396	3.859	4.256	4.578	
Gust (m/s)	0.00	1.820	2.225	2.573	2.869	3.115	3.319	3.491	3.640
	0.10	1.852	2.251	2.592	2.883	3.125	3.326	3.495	3.642
	0.20	1.885	2.278	2.614	2.901	3.139	3.336	3.503	3.649
	0.30	1.923	2.312	2.643	2.924	3.157	3.351	3.515	3.658
	0.40	1.966	2.351	2.676	2.952	3.181	3.372	3.533	3.673
	0.50	2.012	2.393	2.713	2.984	3.209	3.396	3.554	3.692
	0.60	2.061	2.437	2.751	3.017	3.239	3.422	3.578	3.713
	0.70	2.114	2.484	2.794	3.055	3.272	3.453	3.605	3.738
	0.80	2.171	2.532	2.836	3.093	3.307	3.485	3.635	3.765
	0.90	2.230	2.581	2.880	3.133	3.343	3.518	3.665	3.794
1.00	2.290	2.630	2.923	3.172	3.379	3.552	3.697	3.824	

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