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FDTD Computation of Space/Time Integrated Electromagnetic Lagrangian: New Insights into Design of Mutually Coupled Antennas

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Abstract—In this paper, we develop a formalism based on electromagnetic Lagrangian which provides new insights about the near-field reactive energy density around generic antennas for arbitrary spatio-temporal excitation signals. Using electric and magnetic fields calculated via FDTD technique and interpolation routines, we compute the EM Lagrangian density around antennas (thin-wire dipoles and Yagi-Uda Arrays). Spatial maps of time-integrated EM Lagrangian density sheds light onto the capacitive/inductive nature of reactive energy density distribution around antennas and highlight its inherent connection with inter-element mutual coupling.

Index Terms—Electromagnetic Lagrangian, FDTD (finite-difference time-domain), MIMO (multiple-input multiple-output), UWB (Ultra-wideband), Reactive Energy density, Mutual Coupling.

I. INTRODUCTION

The subject of understanding the radiation mechanism of antennas (from accelerated charge and/or time-varying current models) along with the propagating energy flow and reactive (or “stored”) energy dynamics around antennas, has intrigued researchers over many decades [1]–[9]. Consequently, the concepts of electromagnetic inertia, reactive

time, localized waves and energy flow velocity have been developed to shed more light onto the spatio-temporal EM energy dynamics around radiating structures [3], [7]. Also, considerable emphasis has been put on the use of analytical techniques that can generate closed form expressions for fields, energy and associated quantities for well-known current distributions [1], [8].

Very recently, the concept of Poynting localized energy was introduced and applied for antennas modelled in FDTD (finite-difference time-domain) framework, with the aim of evaluating space-time non-propagating energy density around antennas [10]–[11]. It was highlighted in [11] that, the traditional Q-factor approach can indeed assist in quantifying the antenna impedance matching and in determining the *temporal* evolution of total reactive energy in an observation volume around antennas. But the necessity for looking beyond Q-factors and focusing on the *general spatio-temporal* energy dynamics around antennas was also stressed upon in [11]. While the localized energy concept was used in [10] and [11] for the prediction of gain-enhancement in strongly coupled Yagi-Uda type antennas, it did not provide the finer details about the dominant (contributing) fields (electric/magnetic) for energy localization. Furthermore in [11], localized energy density signatures were used to track mutual coupling paths at a preliminary level, a problem that is also addressed by several other research groups by techniques like Poynting streamlines, energy parcels and reaction theorem

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[12]-[14]. However, mutual coupling between multiple antennas needs to be further investigated for possible applications in MIMO systems or electromagnetic interference (EMI) mitigation [15]-[16].

In [17], EM Lagrangian and its intricate connection with the energy flow velocity (also explored in [2], [6]) is revisited. The possible use of EM Lagrangian and flow velocity in assessing the transitions from radiating (forward/backward propagating) to “electrostatic/magnetostatic” field regimes was strongly advocated in [17]. But the numerical examples in [17] mainly use *analytical* space-time current (eg. Exponentially decaying) or field expressions. Note that, analytical solutions to general EM problems (eg. analysis and design of complex shaped printed/wire antennas) are not always feasible, and one needs to use computational EM tools like Finite Element Methods (FEM), Method of Moments (MoM), FDTD etc.

In this paper, we first use time-marching FDTD scheme to obtain the near-zone electric and magnetic fields around antennas with arbitrary port-excitation signal. We then evaluate the EM Lagrangian density around antennas from these FDTD-computed electric/magnetic fields. We further calculate quantities based on spatially or temporally integrated EM Lagrangian density to:

- obtain crucial understanding of the inductive/capacitive reactive energy density distribution around antennas;
- identify regions of mutual coupling and near-field energy exchange;

To the best of the authors’ knowledge, the concepts of spatial or temporal integration of FDTD-computed EM Lagrangian are applied to analyze generic two-port antenna systems for the first time.

II. THEORY OF ELECTROMAGNETIC LAGRANGIAN AND INTEGRATION WITH FDTD

A. Definitions

The EM *Lagrangian density* $L = L(\mathbf{r}, t)$ at any point having space-time varying electric field $\mathbf{E} = \mathbf{E}(\mathbf{r}, t)$ and magnetic field $\mathbf{H} = \mathbf{H}(\mathbf{r}, t)$ in a simple

(linear, isotropic and homogeneous) lossless non-dispersive medium is formally defined as [17]:

$$L = \frac{1}{2}\varepsilon\mathbf{E} \cdot \mathbf{E} - \frac{1}{2}\mu\mathbf{H} \cdot \mathbf{H}, \quad (1)$$

where ε and μ respectively stand for the permittivity and permeability of the concerned medium. Using the *Hamiltonian* or total energy density $u = u(\mathbf{r}, t)$, we can get the normalized Lagrangian $l = l(\mathbf{r}, t)$ as follows:

$$l = \frac{L}{u}, \text{ where } u = \frac{1}{2}\varepsilon\mathbf{E} \cdot \mathbf{E} + \frac{1}{2}\mu\mathbf{H} \cdot \mathbf{H}. \quad (2)$$

As discussed in [17], one can have a simple characterization of radiating and reactive fields using the following mapping of normalized Lagrangian l : (a) $l \rightarrow 0$: Pure, far-field EM wave, (b) $l \rightarrow +1$: Electrostatic field and (d) $l \rightarrow -1$: Magnetostatic field. On the other hand, the *energy flow velocity* $\mathbf{v}_e = \mathbf{v}_e(\mathbf{r}, t)$ at that point is given by [4], [6]:

$$\mathbf{v}_e = \frac{\mathbf{S}}{u} = \frac{\mathbf{E} \times \mathbf{H}}{\frac{1}{2}\varepsilon\mathbf{E} \cdot \mathbf{E} + \frac{1}{2}\mu\mathbf{H} \cdot \mathbf{H}}, \quad (3)$$

Here $\mathbf{S} = \mathbf{E} \times \mathbf{H}$ represents the instantaneous Poynting vector. The normalized energy flow velocity $v = v(\mathbf{r}, t)$ can be further obtained as,

$$v = \frac{|\mathbf{v}_e|}{c}, \text{ where } c = 1/\sqrt{\mu\varepsilon}. \quad (4)$$

Note that, for far-zone propagating EM fields, one would have $v \rightarrow 1$. For the spatial regions having reactive EM fields, one would observe $v < 1$ (i.e. slowing down of energy-flow).

B. EM Lagrangian, Reactive Energy Density, EM Inertia and Flow Velocity

It is important to connect EM Lagrangian density L with the existing concepts of EM inertia, reactive energy and flow-velocity [2], [3]. With this objective, we write from (1) and (2):

$$\begin{aligned} u^2 - L^2 &= \left[\frac{1}{2}\varepsilon|\mathbf{E}|^2 + \frac{1}{2}\mu|\mathbf{H}|^2 \right]^2 - \left[\frac{1}{2}\varepsilon|\mathbf{E}|^2 - \frac{1}{2}\mu|\mathbf{H}|^2 \right]^2 \\ &= \varepsilon\mu|\mathbf{E}|^2|\mathbf{H}|^2 = \frac{1}{c^2}|\mathbf{E}|^2|\mathbf{H}|^2 \end{aligned} \quad (5)$$

Note that, in the above simplification we have used $\mathbf{E} \cdot \mathbf{E} = |\mathbf{E}|^2$ and $\mathbf{H} \cdot \mathbf{H} = |\mathbf{H}|^2$. Now, we recall the vector identity:

$$|\mathbf{E}|^2 |\mathbf{H}|^2 = |\mathbf{E} \times \mathbf{H}|^2 + |\mathbf{E} \cdot \mathbf{H}|^2 \quad (6)$$

Using (6) in (5), and rearranging the terms, we obtain:

$$L^2 = u^2 - \frac{1}{c^2} (|\mathbf{v}_e|^2 u^2 + |\mathbf{E} \cdot \mathbf{H}|^2) \quad (7)$$

Here, $\mathbf{v}_e$ is the energy flow-velocity as discussed in (3). Now, utilizing the concept of *EM inertia density* w_{inertia} from [2] and [3], we can express L as:

$$L = \sqrt{w_{\text{inertia}}^2 - \frac{1}{c^2} |\mathbf{E} \cdot \mathbf{H}|^2}, \quad (8)$$

where,

$$w_{\text{inertia}} = \sqrt{u^2 \left(1 - \frac{|\mathbf{v}_e|^2}{c^2}\right)} = \sqrt{u^2 - \frac{|\mathbf{S}|^2}{c^2}} \quad (9)$$

Traditionally, the space-time *reactive (rest) energy density* w_{reac} around any antenna is obtained by subtracting a radiation power term (based on $\mathbf{S}$) from the total EM energy density u (see [11] for more relevant references on this subject). In [2] and [3], the link between w_{inertia} and w_{reac} associated with any EM wave is explained in great detail. Note that, in the antenna far-field, we have TEM (transverse electromagnetic) plane-wave propagation, implying that: $\mathbf{E} \cdot \mathbf{H} = 0$ and $|\mathbf{v}_e| \rightarrow c$. Consequently we get $L = 0$ and $w_{\text{inertia}} = 0$ in the antenna far-field. In the antenna near-field regime, we have $|\mathbf{v}_e| \neq c$. Furthermore, $\mathbf{E}$ and $\mathbf{H}$ are no longer mutually orthogonal in the close-proximity of radiating antennas (i.e., $\mathbf{E} \cdot \mathbf{H} \neq 0$). Therefore, L in (8) provides an useful measure of the reactive energy density in the antenna near-field.

C. Estimating EM Lagrangian based Quantities from FDTD

Note that, to calculate l from respectively (2), one would require evaluation of $\mathbf{E}$ and $\mathbf{H}$ at same spatial location and time-instant; however due to the staggered nature of FDTD-grid [18]-[20], mathematical operations (i.e. cross-product, summation etc) on instantaneous values of $\mathbf{E}$ and $\mathbf{H}$ cannot be

performed in a straightforward manner [21]. Therefore, one generally resorts to interpolation schemes [21], for evaluation of time-averaged Poynting flow [22] and Poynting localized energy [10]. Once the space-time values of $L = L(x, y, z, t)$ are obtained from a time-marching FDTD routine and suitable interpolation scheme, it is judicious to use temporal and spatial integration via operators $\mathcal{T}\{\star\}$ and $\mathcal{S}\{\star\}$, where:

$$\mathcal{T}\{L\} = \int_0^{T_m} L(x, y, z, t) dt, \quad (10)$$

$$\mathcal{S}\{L\} = \int_{x_{\min}}^{x_{\max}} \int_{y_{\min}}^{y_{\max}} \int_{z_{\min}}^{z_{\max}} L(x, y, z, t) dx dy dz, \quad (11)$$

Here T_m stands for the maximum FDTD computation time, and $x_{\min}$, $x_{\max}$, $y_{\min}$, $y_{\max}$, $z_{\min}$ and $z_{\max}$ represent the spatial boundaries of the FDTD simulation space. The FDTD parameters values have to be specified such that the simulated multi-port antenna system metrics (eg. S-parameters, correlation coefficient etc, see [23]) are accurate, and can be independently verified from other simulation techniques, e.g., MoM (Method-of-moments) in [24].

Once the quantities L and u are computed from FDTD electric and magnetic fields, we determine the quantities $\bar{l}$ and $\langle L \rangle$ using the approach of [4] as follows:

$$\bar{l} = \frac{\mathcal{T}\{L\}}{\mathcal{T}\{u\}} \text{ and } \langle L \rangle = \frac{\mathcal{S}\{L\}}{\max[|\mathcal{S}\{L\}|]} \quad (12)$$

Since the ratio of the average is not equal to the average of the ratios, $\bar{l}$ in (12) is not equal to $\mathcal{T}\{l\}$ where $l = L/u$. However, it can be shown that the inequalities $-1 \leq \bar{l} \leq 1$ and $-1 \leq \langle L \rangle \leq 1$ hold for any antenna system facing a finite duration temporal excitation (rigorous proof to be provided elsewhere). Note that, the impedance bandwidth and mutual coupling characteristics of any multi-antenna system are fundamentally related to the near-field reactive energy density, which in turn is inherently linked with the the EM Lagrangian density L (see section II-B). Therefore, $\bar{l}$ and $\langle L \rangle$, which are obtained via further post-processing on L , can provide valuable insights regarding the space-time dynamics of antenna reactive energy density.

The quantity $\langle L \rangle$ is computed by integrating L over an observation volume and consequent normalization. Therefore, $\langle L \rangle$ provides an instantaneous measure of the near-field reactive energy in a volume circumscribing the antenna under consideration. If $\langle L \rangle \rightarrow +1$, one can comment that near-field reactive energy is capacitive in nature. On the other hand, the inductive nature of the near-field reactive energy is confirmed if $\langle L \rangle \rightarrow -1$. Now in [25], it is demonstrated that narrowband high Q-antennas have more temporal fluctuations in the reflected as well as received voltage signal, while for wide-band low Q-antennas these signal levels die down very fast. It is also well-known in circuit theory that Q-factor is closely related to the temporal decaying of the stored energy levels, i.e., the total reactive energy in the volume under observation will decay much quickly for a low-Q system. Since $\langle L \rangle$ provides an estimate of the total reactive energy in an observation volume around an antenna, less temporal fluctuations and quicker decay of $\langle L \rangle$ for an antenna excited by short-pulse would imply that reduction in system Q-factor (i.e., increase in the impedance bandwidth).

The other Lagrangian-based quantity $\bar{l}$ quantifies the spatial distribution of reactive energy density around antennas, something that is not available from the traditional Q-factor approach. The regions around antenna having $\bar{l} \rightarrow 1$ indicates greater capacitive reactive energy density. Similarly, if $\bar{l} \rightarrow -1$, one can say that inductive reactive energy density is higher in those regions. As one moves to the antenna far-field, $\bar{l}$ approaches zero value, as expected. We will also demonstrate through numerical examples that the spatial maps of $\bar{l}$ for different temporal excitation signals reveal the mutual coupling “links” between closely spaced antennas.

III. RESULTS OF NUMERICAL COMPUTATION ON COUPLED ANTENNA SYSTEMS

A. Two-Port System of Narrow-band Dipoles

Fig. 1(a) shows the schematic diagram of a thin-wire center-fed dipole-based two-port configuration placed in a FDTD framework based on Cartesian co-ordinates. We use rectangular grid size of $\Delta x = \Delta y = \Delta z = \Delta = 1$ mm and total problem space as

Fig. 1. (a) Schematic diagram of the two dipole configuration, with $L_{dip} = 36$ mm and $d = 16$ mm (Delta feed-gap shown in zoomed view). (b) Simulated S-parameter magnitude computed using in-house FDTD and MATLAB Antenna toolbox.

$70\Delta \times 70\Delta \times 70\Delta$, by imposing Berenger’s PML (Perfectly matched layers) along each face on a 10Δ thick boundary. Following the CFL limit for stability [20], we choose $\Delta t = 1.588$ ps. The thin-wire PEC dipoles are implemented by enforcing the z -component of electric field to be zero in the required locations. We first estimate the S-parameters of two-dipole system of Fig. 1(a), by exciting the dipoles-1 and 2 by using differentiated Gaussian pulse $p_{dg}(t)$ as:

$$p_{dg}(t) = -\frac{(t - \tau_1)}{\sigma_1^2} \exp\left[-\frac{(t - \tau_1)^2}{2\sigma_1^2}\right], \quad (13)$$

with $\tau_1 = 100\Delta t$, $\sigma_1 = 10\Delta t$. To feed the dipoles, “Delta-gap” model is used [20], and when one dipole is excited by the temporal signal, the other one is connected to a matched load resistor in the Delta-gap region. Total simulation time $T_m =$

$3000\Delta t$ is chosen, so that the transient signals due to pulse excitation die out significantly and post-processing is not erroneous or misleading. Using the procedure described in [23], the S-parameters are calculated and benchmarked (see Fig. 1(b)) with respect to MATLAB Antenna toolbox based on MoM [24]. Fig. 1(b) suggests that the two-dipole system shows narrow-band resonant characteristics with $|S_{11}| = -29.18$ dB at 3.86 GHz and $|S_{11}| \leq -10$ dB in the range 3.66 – 4.01 GHz. Also, high mutual coupling ($|S_{21}| = -6.86$ dB) is observed as the separation between the two dipoles is 0.206λ , where λ = free-space wavelength at 3.86 GHz).

Fig. 2. Simulated variation of: (a) $\langle L \rangle$ with time, (b) $\bar{l}$ along the yz -plane containing the dipoles for the system of Fig. 1(a). Dipole-1 is excited for modulated Gaussian pulse (see (14)) with $f_0 = 2$ GHz. Also for clarity, images of the two dipoles are superimposed in spatial maps of $\bar{l}$.

Next, we consider a modulated Gaussian pulse (typically considered in indoor channel-modelling problems, see [22]) given by:

$$p_{\text{mg}}(t) = \exp\left[-\frac{(t - \tau_2)^2}{2\sigma_2^2}\right] \sin(2\pi f_0 t), \quad (14)$$

where $\tau_2 = 600\Delta t$, $\sigma_2 = 200\Delta t$. When dipole-1 is excited by $p_{\text{mg}}(t)$ with $f_0 = 2.0$ GHz, we have $0 \leq \langle L \rangle \leq 1$ most of the time (Fig. 2(a)). This indicates that at 2.0 GHz the reactive energy density distribution around the antenna system is predominantly capacitive (i.e. $\bar{l} \rightarrow +1$, especially near the arm tips see Fig. 2(b)), with very less spatial region having inductive nature (i.e. $\bar{l} \rightarrow -1$). This is anticipated as for the system of Fig. 1(a), we are essentially operating in the electrically small antenna regime ($ka = 0.825 < 1$) at $f_0 = 2.0$ GHz, where $k = 2\pi f_0/c$ and $a = \sqrt{(L_{\text{dip}}/2)^2 + (d/2)^2}$

is radius of the smallest sphere (see Fig. 1(b)) circumscribing the two-dipole system.

Fig. 3. Simulated variation of: (a) $\langle L \rangle$ with time, (b) $\bar{l}$ along the yz -plane containing the dipoles for the system of Fig. 1(a). Dipole-1 is excited for modulated Gaussian pulse (see (14)) with $f_0 = 4$ GHz. Also for clarity, images of the two dipoles are superimposed in spatial maps of $\bar{l}$.

Fig. 4. Simulated variation of: (a) $\langle L \rangle$ with time, (b) $\bar{l}$ along the yz -plane containing the dipoles for the system of Fig. 1(a). Dipole-1 is excited for modulated Gaussian pulse (see (14)) with $f_0 = 6$ GHz. Also for clarity, images of the two dipoles are superimposed in spatial maps of $\bar{l}$.

When f_0 is chosen to be 4 GHz, which is very close to the dipole impedance matching point (see Fig. 1(b)), the system of Fig. 1(a) behaves as a narrow-band coupled dipole. Hence the system reactive energy goes back-and-forth between inductive and capacitive nature when the excitation frequency is $f_0 = 4$ GHz. This is corroborated from the fact that $\langle L \rangle$ oscillates in equal manner above 0-value, within the range of $[-1, 1]$ (Fig. 3(a)). For $f_0 = 4$ GHz, one can observe $\bar{l} \rightarrow +1$ signifying high concentration of capacitive reactive energy density near the dipole-tips (Fig. 3(b)). On the other hand, the region between the two dipole centers demonstrate strong complementary inductive energy density concentration with $\bar{l} \rightarrow -1$ (Fig. 3(b)). The spatial maps of $\bar{l}$ further illustrate *energy-*

exchange links between the two side-by-side dipoles (Fig. 3(b)), which is indicative of strong inter-dipole mutual coupling at $f_0 = 4$ GHz.

Finally, as f_0 is increased to 6 GHz, we observe asymmetric temporal variation of $\langle L \rangle$ above the zero-level. This implies that fundamental dipole resonant mode is not excited at this frequency. Also the mutual coupling $|S_{21}|$ is much less at 6 GHz, as compared to that at 4 GHz (see Fig. 1(b)). This reduced mutual coupling phenomenon is reflected in Fig. 4(b) as well, where the spatial maps of $\bar{l}$ around the individual dipoles for $f_0 = 6$ GHz no longer show a *connected characteristics* like the case with $f_0 = 4$ GHz (see Fig. 3(b)).

Fig. 5. (a) Modified two-dipole system with reflectors and directors having $L_{dir} = 34$ mm, $L_{ref} = 40$ mm and $x_{off} = 15$ mm (rest same as Fig. 1(a)), and (b) frequency variation of simulated S -parameters.

B. Two-port System of Yagi-Uda Arrays

We further apply the EM Lagrangian approach on a modified two-dipole system (see Fig. 5(a)) with one reflector and one director. Using differentiated

Gaussian pulse excitation of (13), we first compute the modified wide-band S-parameter response of the two-port array of Yagi-Uda elements as shown in Fig. 5(a). It is worth noting that due to the presence of additional reflector and directors, the mutual coupling level has considerably reduced near the operating frequency of 4 GHz (see Fig. 5(b)). At the same time, Fig. 5(b) shows slight degradation in the impedance matching performance for the Yagi-Uda array system of Fig. 5(a), as compared to the dipole based configuration of Fig. 1(a).

Fig. 6. (a) Variation of $\langle L \rangle$ with time for the systems of Fig. 1(a) and Fig. 5(a), (b) For the system of Fig. 5(a), distribution of $\bar{l}$ along yz -plane containing the two exciter dipoles (For clarity, images of the exciter dipoles are superimposed in spatial maps of $\bar{l}$). For both cases, we consider dipole-1 to be excited by modulated Gaussian pulse (see (14)) with $f_0 = 4$ GHz.

Next, we compute both $\bar{l}$ and $\langle L \rangle$ for the two-port system of Fig. 5(a), by exciting dipole-1 with $p_{mg}(t)$ of $f_0 = 4$ GHz. Fig. 6(a) shows that in presence of additional reflectors and directors, the $\langle L \rangle$ decays slowly for the Yagi-Uda system

compared to the two-dipole configuration (similar effect was observed in the temporal plots of total reactive energy, see [9] for details).

Note that, the two-dipole configuration of Fig. 1(a) has very high mutual coupling around 4 GHz, which is also reflected in the $\bar{l}$ distributions of Fig. 3(b). For similar modulated Gaussian pulse excitation with $f_0 = 4$ GHz, the reactive energy density distribution around the non-excited dipole for the Yagi-Uda system is significantly reduced (see Fig. 6(b)). This is consistent with the low inter-element coupling observed for the Yagi-Uda system around 4 GHz (see Fig. 5(b)).

IV. DISCUSSIONS

It is well known in physics that the *functionals* used to describe systems in nature generally comprise of an integral over time of a function called the Lagrangian¹. By using *calculus of variations*, it is possible to define the notions of functional and Lagrangian rigorously and apply them to electromagnetism with the aim of deriving the equations of motion in an electromagnetic field, as well as the Maxwell's equations [26]-[28]. However, these rigorous analyses from physics perspective are outside the scope of this manuscript, where the main focus is on the calculation of space-time EM Lagrangian density L from the FDTD-computed electric and magnetic fields.

From all the FDTD simulation results in the previous sections, it is evident that both the quantities $\langle L \rangle$ and $\bar{l}$ derived from Lagrangian bear a fundamental connection with the near-field reactive energy density around antennas, and thereby both impedance matching and mutual coupling parameters². Through spatial maps of $\bar{l}$, it is possible to visually observe the capacitive/inductive energy distribution around antennas in general for arbitrary temporal excitations, something that is not found from classical reactive energy or Poynting localized energy approach (see [10] or [11] for more discussions on this). Robust FDTD analysis of space-

time EM Lagrangian can also play pivotal role in: (i) minimization of the EM interference effects on electronic circuit components near a radiator, sensors, near-field wireless power transfer (WPT) applications.

V. CONCLUSION

We utilize FDTD computed $\mathbf{E}$ and $\mathbf{H}$ fields along with suitable interpolation routines to compute EM Lagrangian based quantities, $\langle L \rangle$ and $\bar{l}$, around arbitrary antenna systems for any temporal excitation. Note that, $\mathbf{E}$ and $\mathbf{H}$ fields in close proximity to antennas have both propagating and non-propagating components, hence their simple plots do not give idea about the distribution of pure reactive energy density [5]. Though the traditional circuit-theoretic Q-factor approach gives some idea about time-variation of the total reactive energy in a given volume, the spatial features of reactive energy density around antennas is not emphasized much. While, the method of [10] can quantify regions with high Poynting localized energy density, the proposed EM Lagrangian approach using $\bar{l}$ provides a unique way to spatially distinguish regions having inductive and capacitive reactive energy density.

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¹In [28], Lagrangian is vaguely referred to as the 'activity' or 'liveliness' of the system.

²For a single-port antenna, this claim can be rephrased as both $\langle L \rangle$ and $\bar{l}$ bears a connection with impedance matching $|S_{11}|$.

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